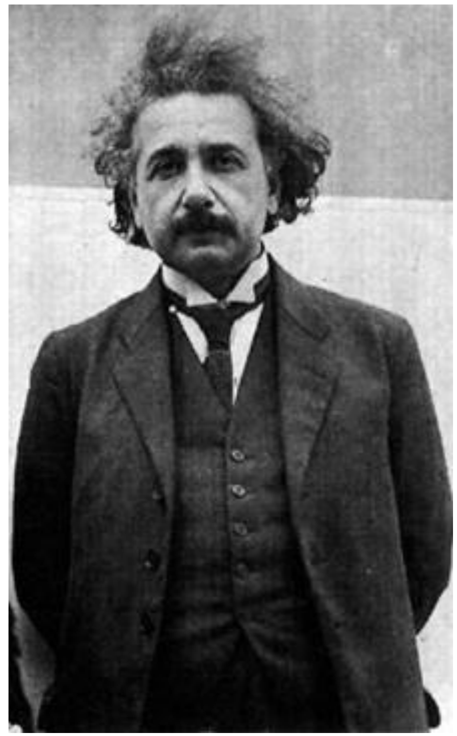
Einstein’s Space and Time

Re-examining the Obvious

“Familiar things happen, and mankind does not bother about them. It requires a very unusual mind to make an analysis of the obvious."

Alfred North Whitehead



Albert Einstein
1879-1955

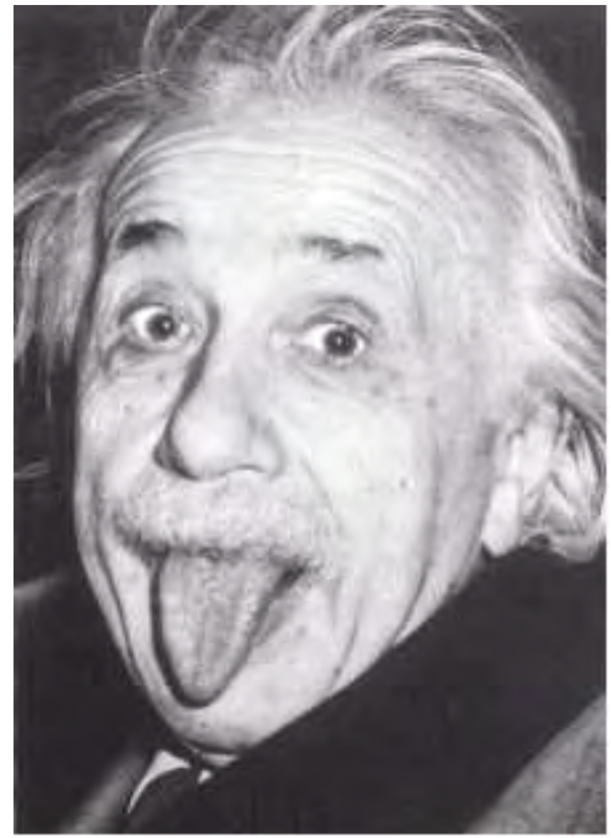
What is Einstein famous for?

- Relativity. Two flavors:

Special Relativity: (SR)	**New notions of space and time, $E=mc^2$. (1905)**
General Relativity: (GR)	**Gravitation, curved Spacetime. (1915)**

- Photons of Quantum Physics (1905)

- Stereotypic eccentric science professor (circa 1950)



What does a relativity concept entail?

- The rules of how nature behaves are unaltered by one’s “viewpoint,” or reference frame.
- In particular, regarding motion, no motion can be said to be an absolute motion.

Relativity

- A relativity principle for motion existed long before Einstein.

Galileo Galilei (1564-1642)

Galileo frequently emphasized the relativity of motion.

- Commonly, the notion of a relativity principle <u>for motion</u> is natural to accept by most everyone.

- **Experience with adjacent moving trains or buses**
- **Motion of the sun, stars and moon in the sky**

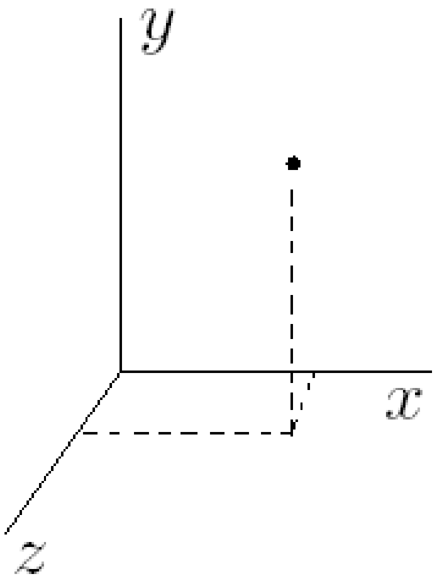
What has relative motion to do with space and time?

Space and time are intimately tied to notions of motion, since motion is itself defined by **where** and **when** events occur.

Consequently, the introduction of reference frames for describing motion will be quite relevant in this discussion.

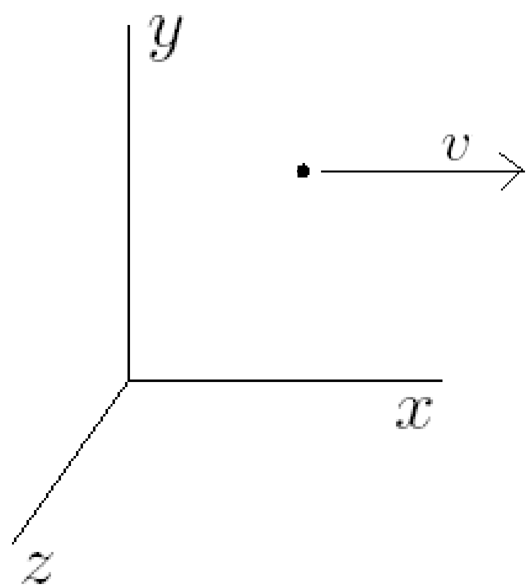
A Reference Frame

Time



Spatial
Coordinates

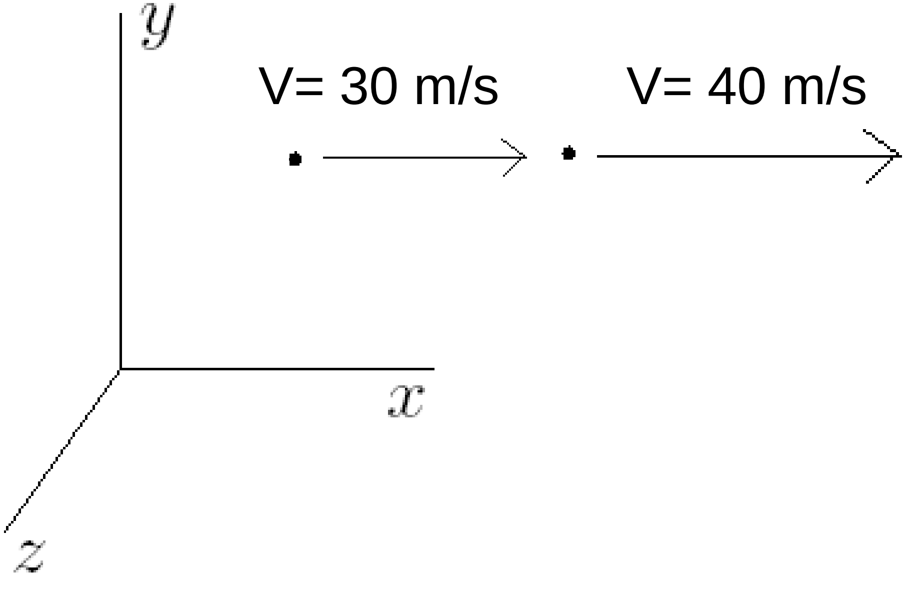
Uniform Motion



Say, v=30 m/s

Constant, unchanging velocity

As compared to non-Uniform Motion



Acceleration: Characterizes <u>changing</u> velocity

Law of Inertia

What is required to keep an object at rest?

Law of Inertia

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Nothing.

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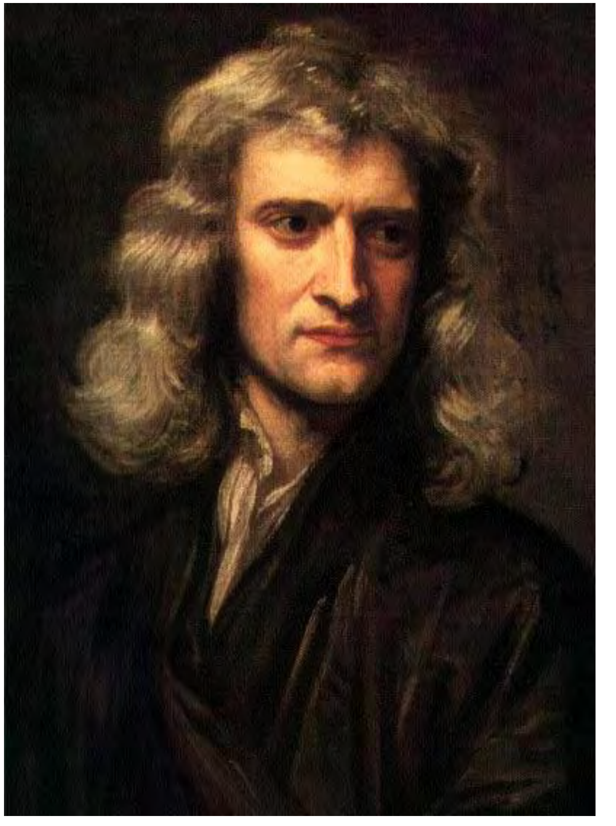
Inertial motion is natural motion

(Seems like the least ad-hoc assumption)

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$F = ma$$

$$\frac{F}{m} = a$$



Sir Isaac Newton

(1642-1727)

Interactions (i.e. forces) cause deviation from inertial (natural) motion

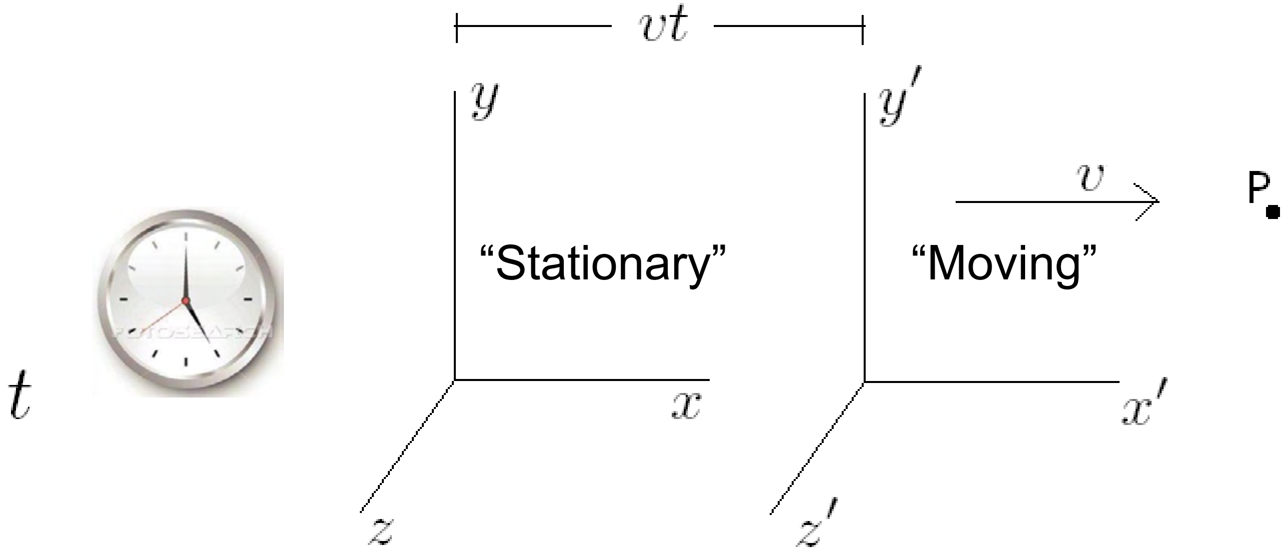
Acceleration is then important in describing the forces. That is, describing the physics.

However:

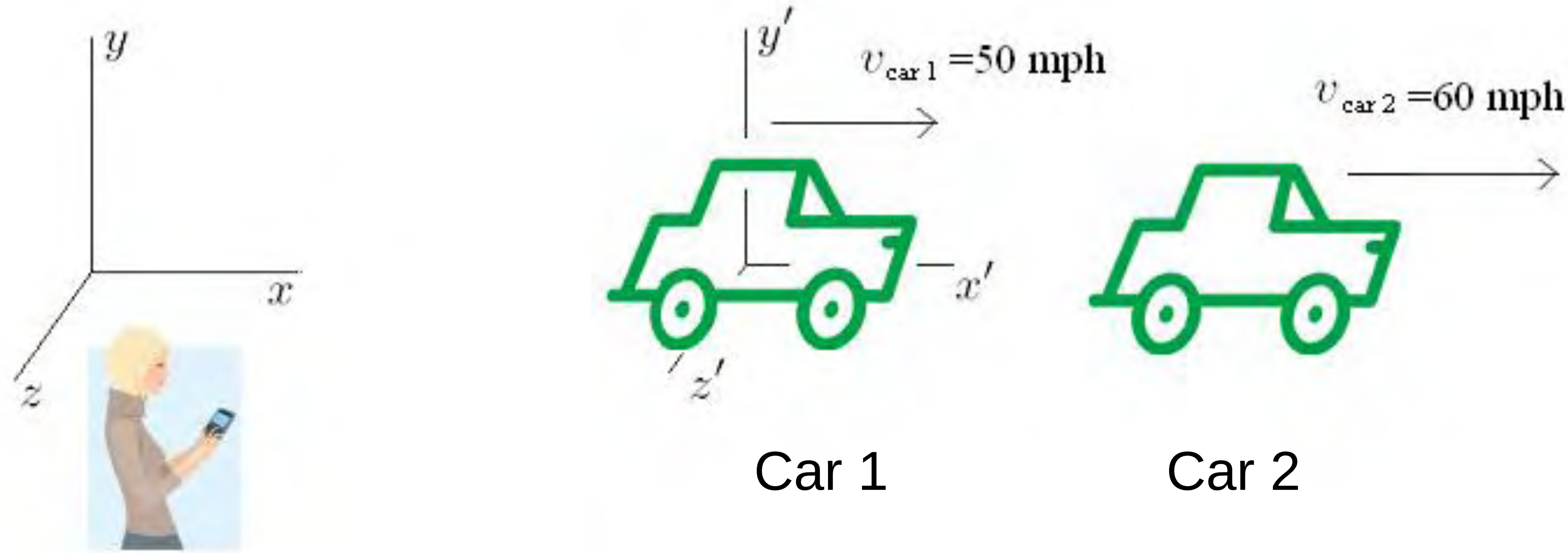
Objects within a slowing car accelerate forward with respect to the car. But, in this case there is no agent of force.

Apparently the law of inertia does not hold in an accelerating frame.

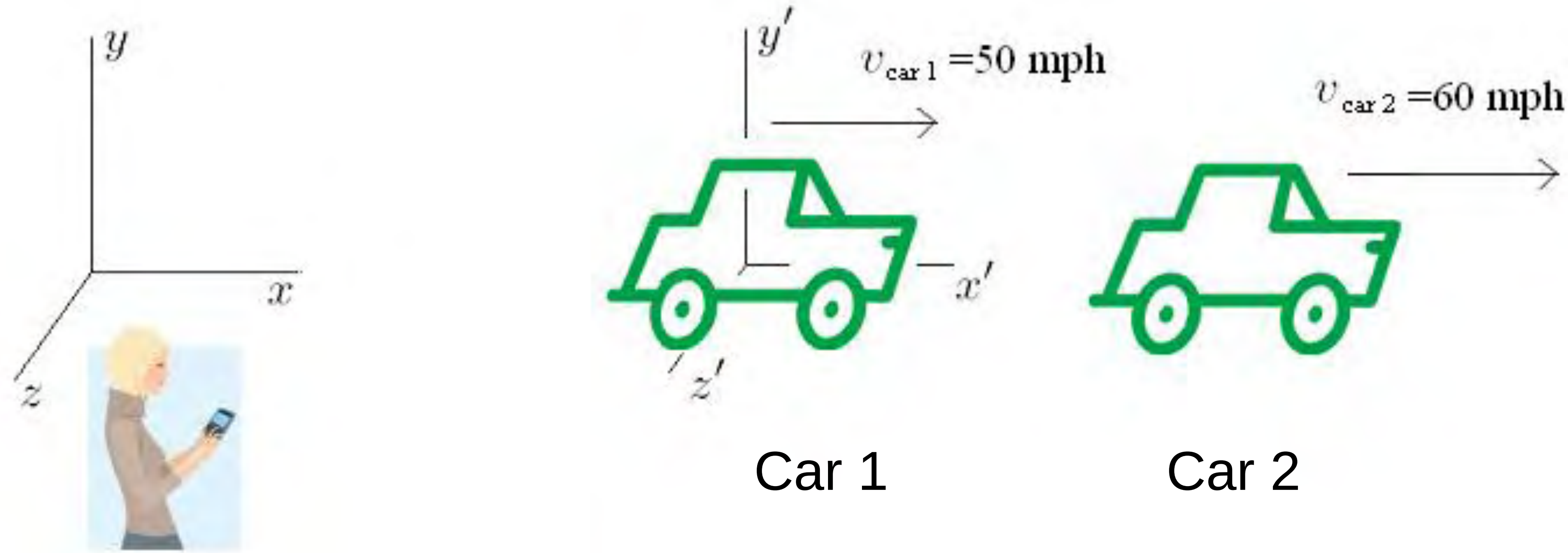
Those frames of reference in which the Law of Inertia does hold are called inertial frames of reference.



Two inertial (non-accelerating) frames of reference observing the point P.

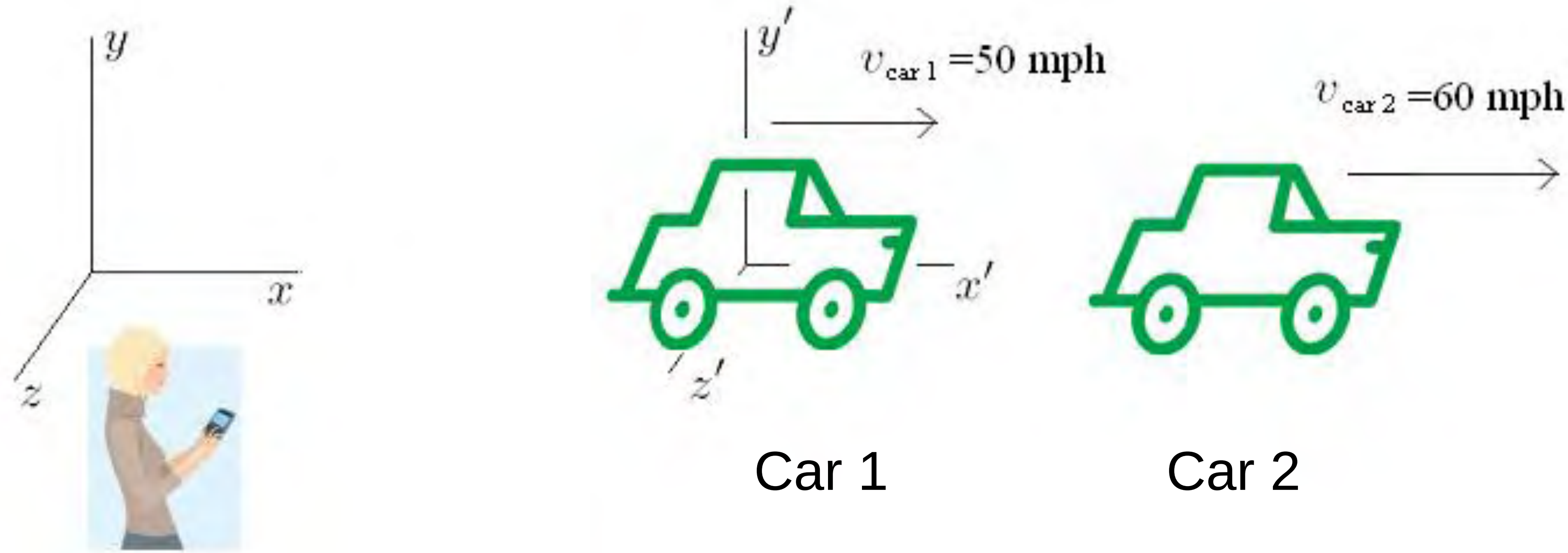


This “stationary” observer sees car 1 and car 2 moving at 50 mph and 60 mph, respectively.



This “stationary” observer sees car 1 and car 2 moving at 50 mph and 60 mph, respectively.

What does the driver in car 1 say about the <u>relative</u> speed of car 2?



This “stationary” observer sees car 1 and car 2 moving at 50 mph and 60 mph, respectively.

What does the driver in car 1 say about the relative speed of car 2?

60 mph - 50 mph= 10 mph

Again, from which frame(s) of reference does the law of inertia hold?

Again, from which frame(s) of reference does the law of inertia hold?

Answer:

Those that are not accelerating frames.

Again, from which frame(s) of reference does the law of inertia hold?

Answer:

Those that are not accelerating frames.

Accelerating with respect to what?!!

But, from which frame(s) of reference does the law of inertia hold?

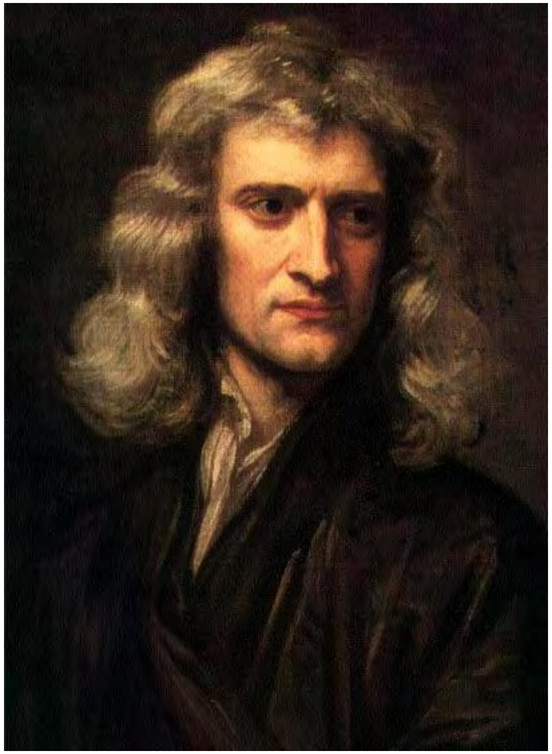
Answer:

Those that are not accelerating frames.

Accelerating with respect to what?!!

Therein lies the problem!

Sir Isaac Newton



Proposed an Absolute Space and Absolute Time

“*Absolute space, <u>in its own nature</u>, <u>without regard to anything external</u>, remains always similar and immovable.*”

“Absolute, True, and Mathematical Time, <u>of itself</u>, <u>and from it own nature</u> flows equably <u>without regard to any thing external</u>”

Sholium to the Principia, 1713

Newton conjectured that Absolute Space “acted” on matter to produce a resistance to changing motion, that Absolute Space in this way would impose inertial behavior.

Newton's critics regarding Absolute Space

(The early "relativists")

- Space exists only as a relation between objects, and which itself has no existence apart from the existence of those objects.
- How can Absolute Space act on something without being itself acted upon? How does one directly detect this Absolute Space?

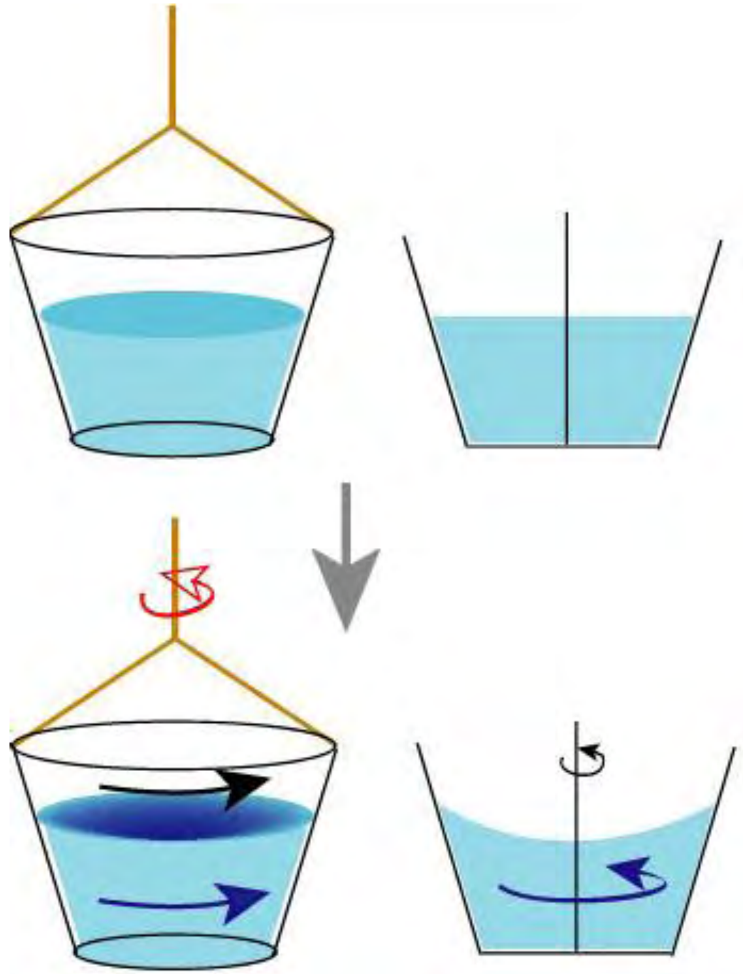
Gottfried Leibniz
1646-1716

George Berkeley
1685-1753

Library of Congress

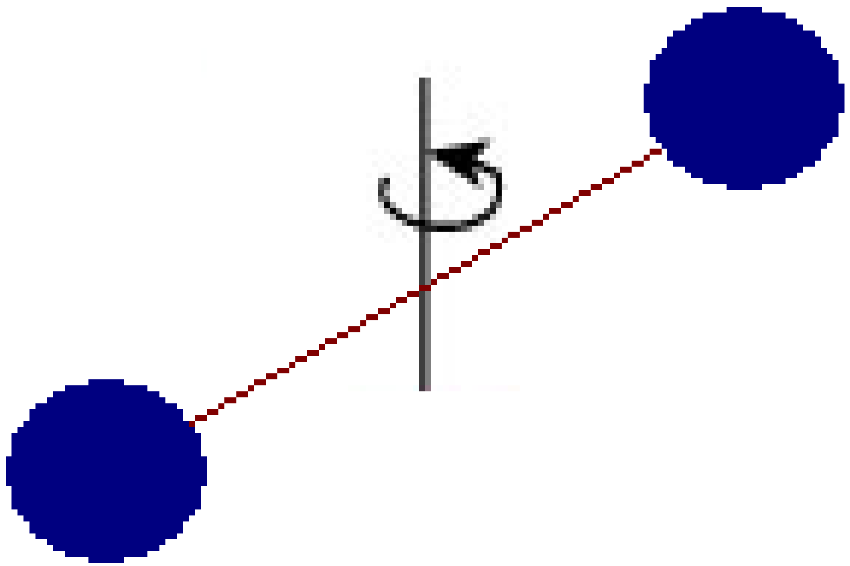
Christian Huygens
1629-1695

Newton's Bucket



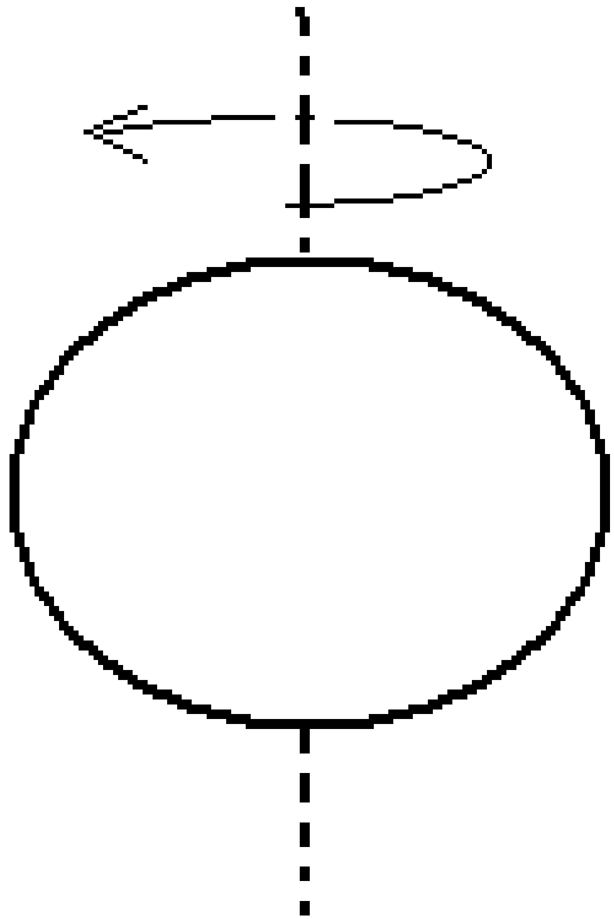
How does the water "know" to curve?
Why can't the frame of the bucket define non-rotation?
Newton: Because the bucket rotates with
respect to Absolute Space

Two “revolving” spheres attached with a rope



How does the rope “know” to be in tension?

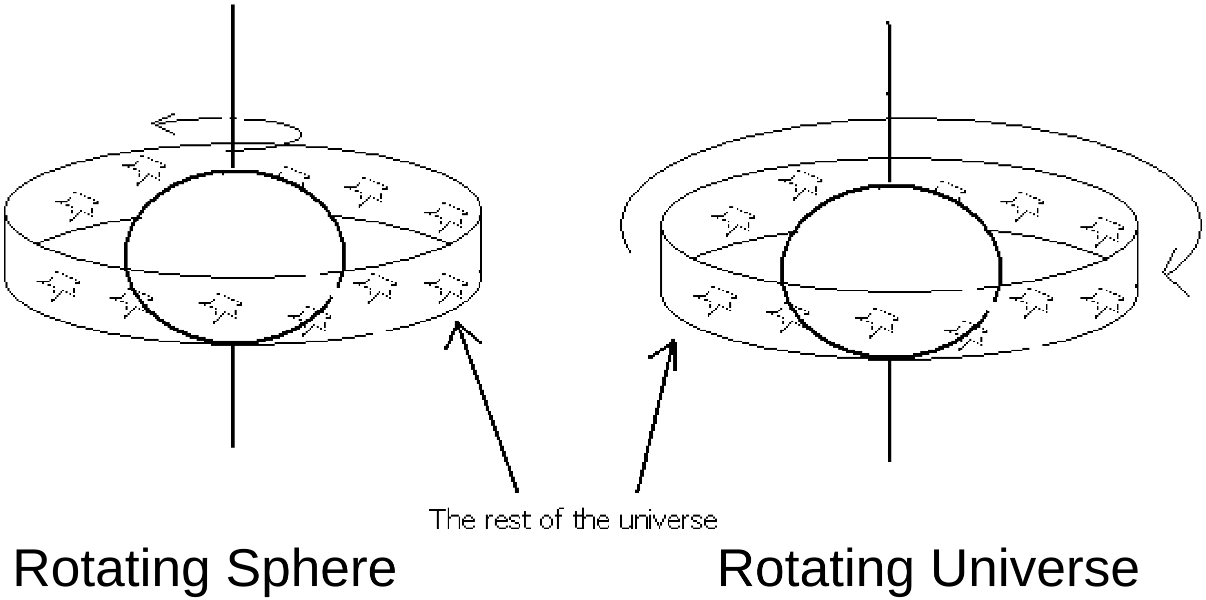
How does the sphere in empty space “know” to bulge?



Why can’t axes fixed to the body of the sphere be said to be non-rotating? Newton: The sphere bulges, thus the sphere is rotating with respect to Absolute Space.

The Absolute Space controversy was a vexing dilemma, but Newtonian mechanics worked so well that most ignored the issue.

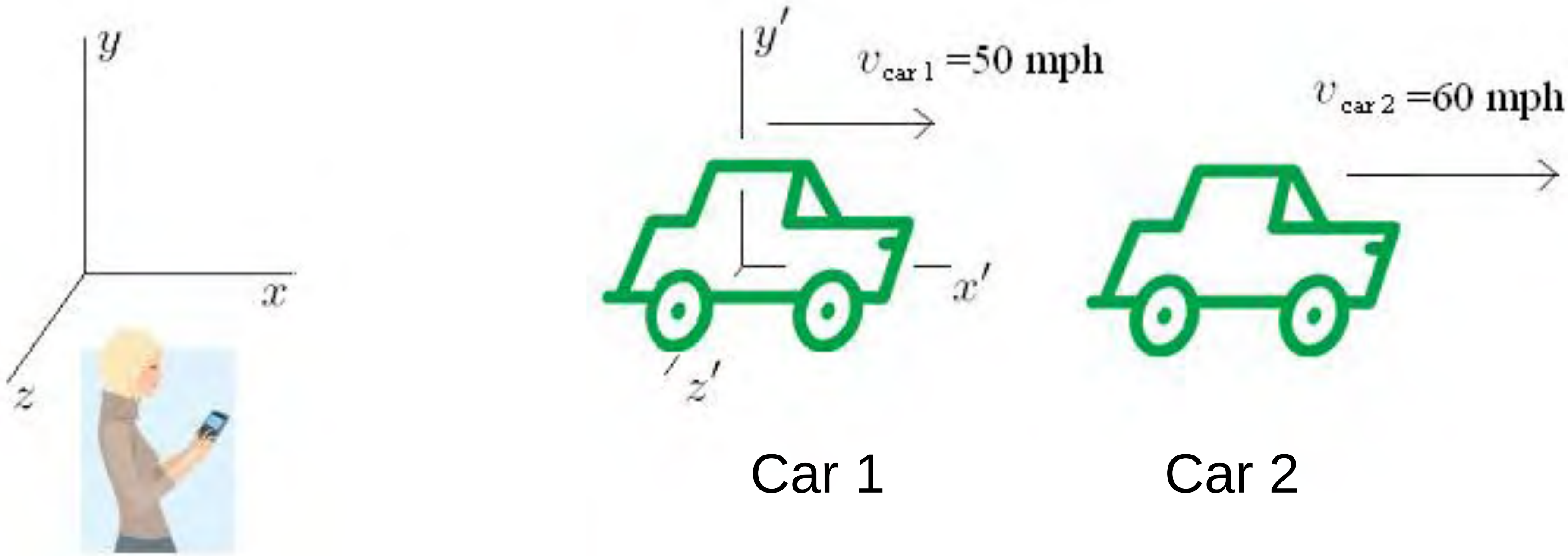
Later, Ernst Mach advanced the relativists rebuttal to Newton's bucket.



Ernst Mach
1838-1916

"The law of inertia must be so conceived that exactly the same thing results from the second supposition as the first."

In both cases the sphere bulges due to the same <u>relative</u> motion



This “stationary” observer sees car 1 and car 2 moving at 50 mph and 60 mph, respectively.

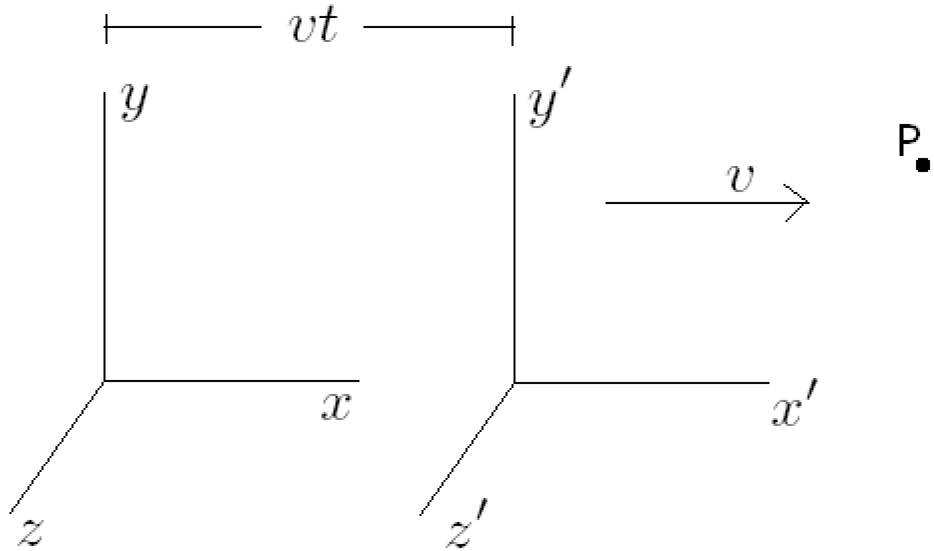
According to our “stationary” observer, in 1 hour car 2 has traveled 60 miles, while car 1 has traveled 50 miles, a difference of 10 miles “Clearly then,” *after one hour* the driver of car 1 observes car 2 to be *10 miles further*. Hence we would think, “obviously” the driver of car 1 concludes that car 2 travels 10 miles in one hour.

Common Assumptions

(based on the views of the unprimed “stationary” observer)

- One hour has passed for <u>both</u> the “stationary” observer, <u>and</u> the driver of car 1.
- The “stationary” observer <u>and</u> the driver of car 1 observe the **same** <u>spatial distance</u> <u>*between*</u> the two cars.

Einstein ultimately reexamines and <u>questions</u> these “obvious” assumptions



Galilean Space and time transformation

Embodies our common assumptions about space and time

$$x' = x - vt$$

$$t' = t$$

- The “stationary” observer and the driver of car 1 observe the **same** <u>spatial distance</u> <u>*between*</u> the two cars.
- One hour has passed for both the “stationary” observer, and the driver of car 1

Galilean relativity **<u>for motion</u>** seemed fine in the late 19thcentury. With it, Newton’s laws of motion were the same in all inertial reference frames. But physics is not only about mechanics and motion.

- Heat
- Electricity and magnetism
- Light and sound
- Waves

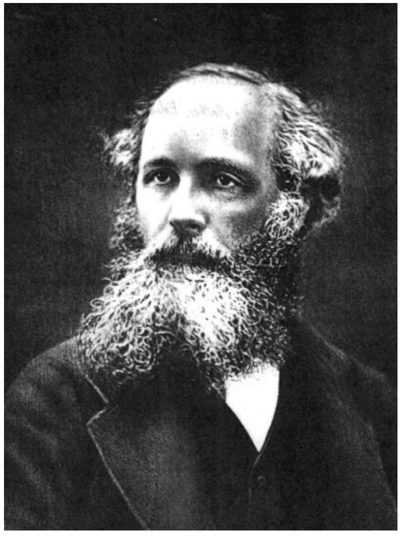
However:

Using Galilean relativity, the very successful laws of electromagnetism didn’t seem to follow a relativity principle!

Maxwell's synthesis and consolidation of the laws of electromagnetism was a great triumph of physics. (1865)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

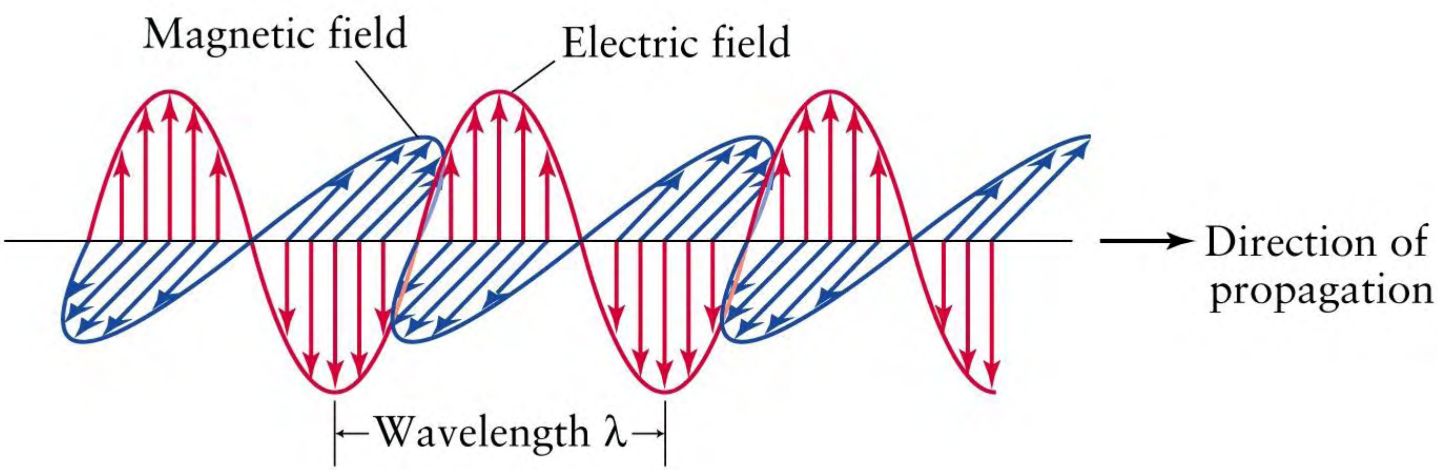
Maxwell's Equations of Electromagnetism

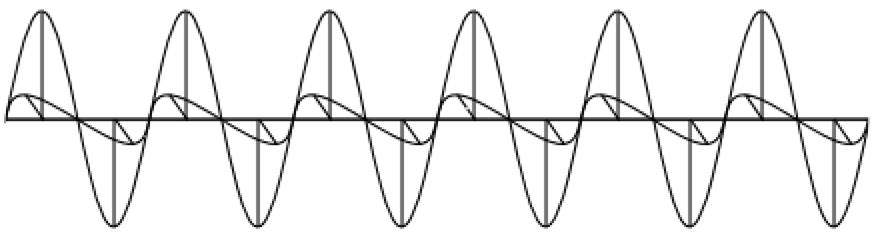


James Clerk Maxwell (1831-1879)

Electric and magnetic effects were thoroughly understood as a unified phenomenon.

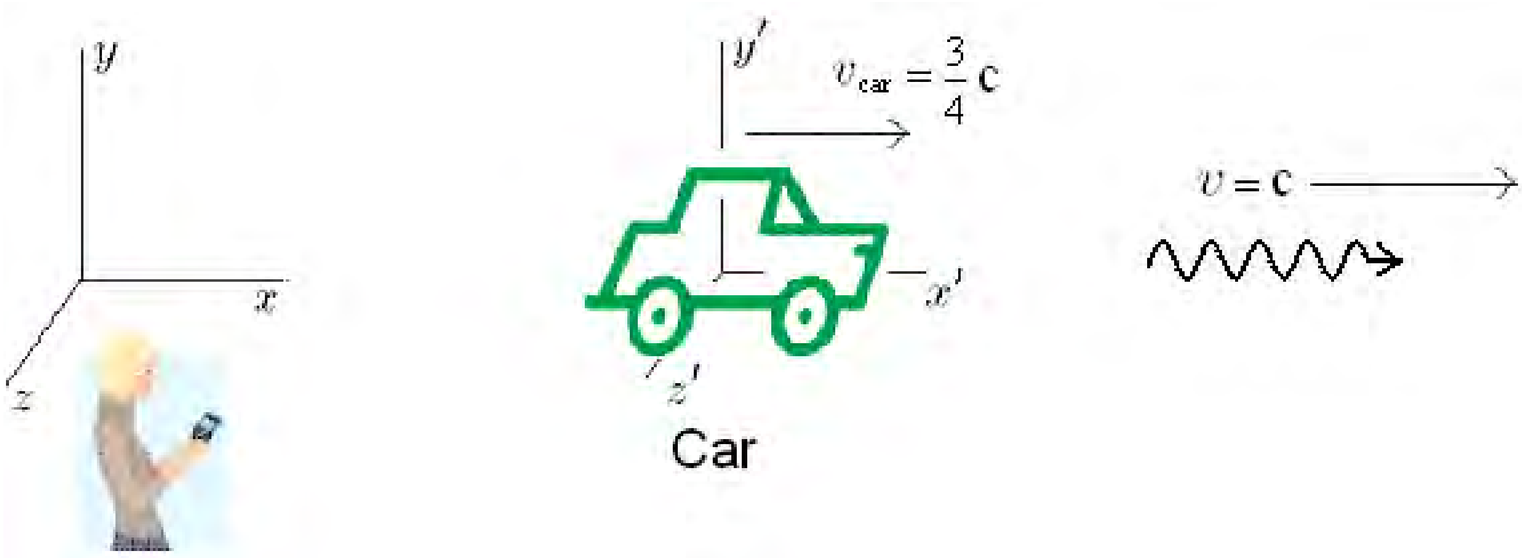
Electric and Magnetic Waves





Maxwell's equations predict propagating waves consisting of electric and magnetic fields which traveled in vacuum at the speed of 3 10^8 m/s (typically denoted by c), the known speed of light.

Dilemma: Speed of light=3 × 10^8 m/s with respect to which inertial frame of reference?



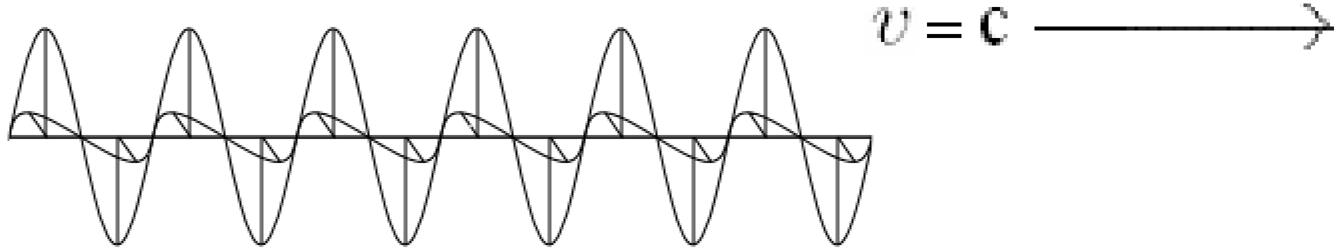
Galilean Relativity would say that the car driver measures the light speed to be ¼ the speed c.

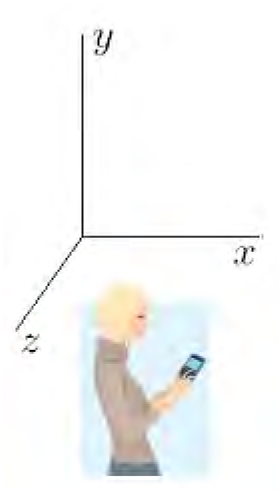
Late 19th Century Crisis in Physics

A young Einstein was influenced by the ideas of Ernst Mach, and what Einstein called Mach's principle.

Einstein felt that **all** the laws of physics should follow a relativity principle

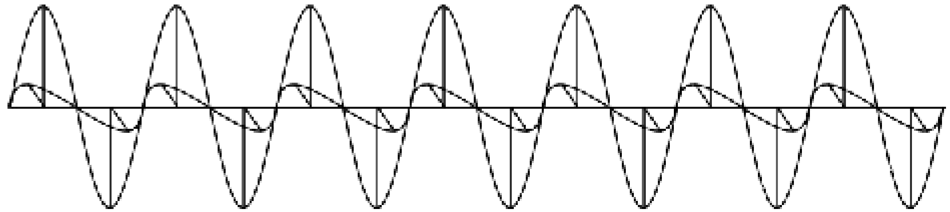
As a teenager Einstein had imagined:

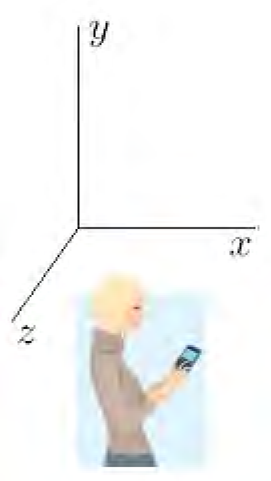


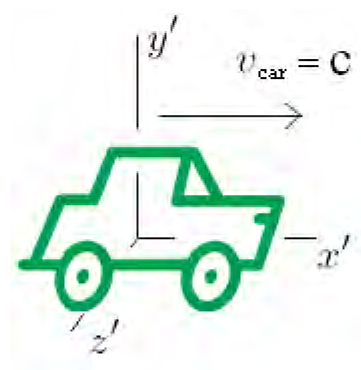


Watching light waves pass

Then Einstein imagined:

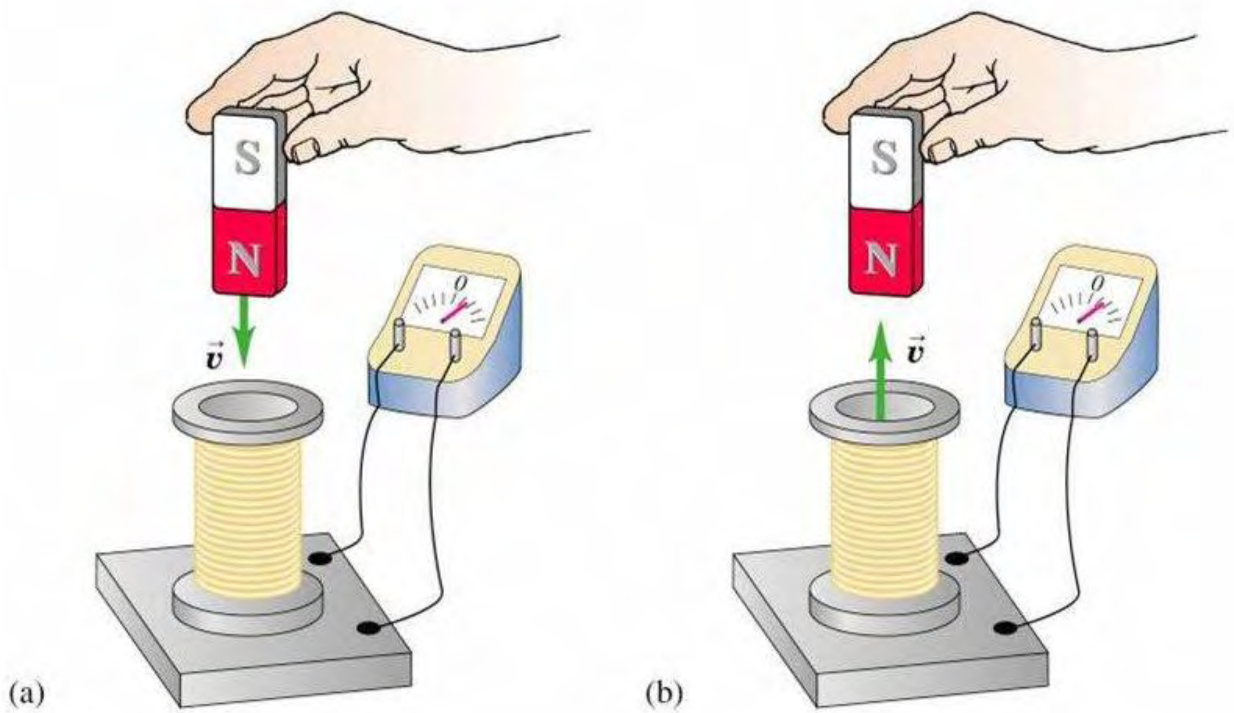






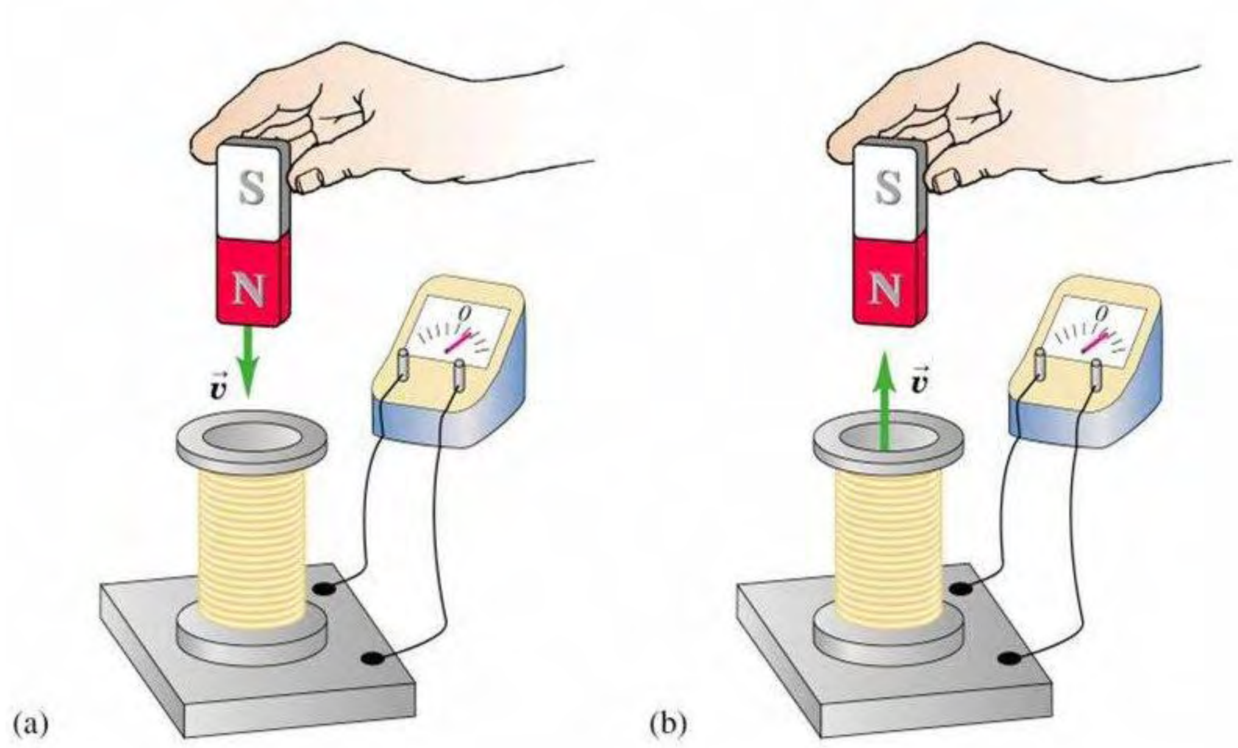
Traveling with the light waves

Significant to Einstein:



(a) (b)

Significant to Einstein:

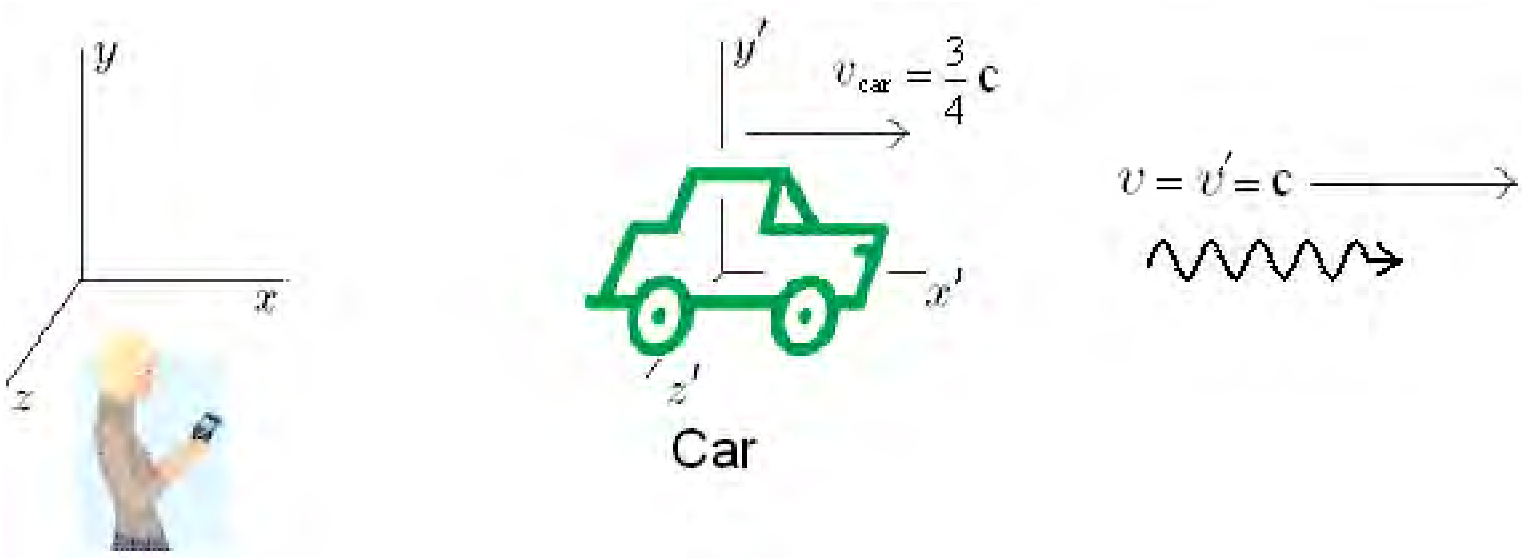


Even though the equations of electromagnetism were <u>in</u>consistent with a relativity principle that uses the <u>Galilean</u> space and time transformation, they still appeared to predict perfectly correct **physical** phenomena from different frames of reference.

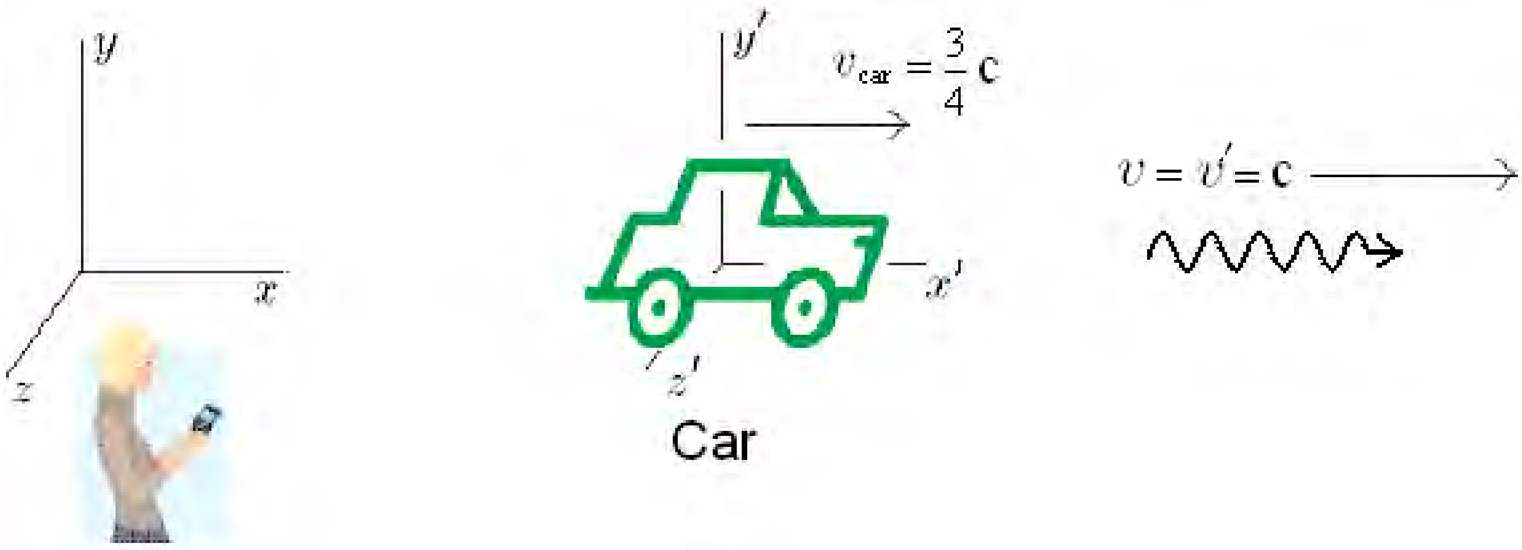
To address his sense that **all** of physics should abide by a relativity principle, Einstein introduces two postulates:

- ALL physical laws viewed from any inertial reference frame are identical (i.e. even the laws of electromagnetism.)
- The speed of light is the same in all such inertial reference frames

Then Einstein proceeded to relentlessly follow the logical consequences



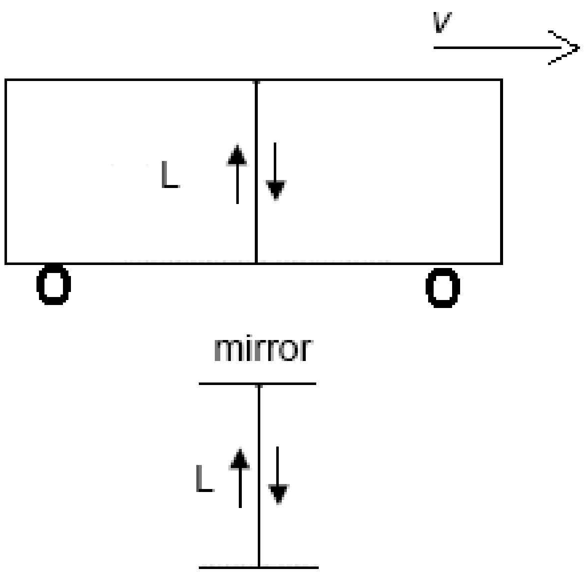
Einstein's relativity instead asserts that the car driver measures the light speed to be c also! The same speed reported by the stationary observer!



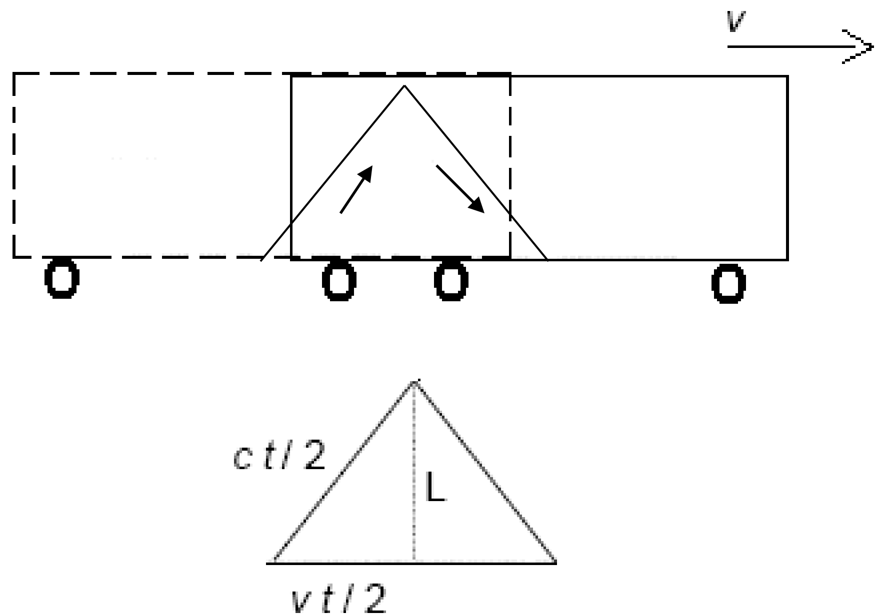
$$c = \frac{\text{(distance traveled)}}{\text{(corresponding time interval)}} = \frac{\text{(distance traveled)}'}{\text{(corresponding time interval)}'}$$

With just the assertion that the speed of light is a constant in all inertial frames, one is seemingly compelled to conclude that time intervals, distance intervals, or both are observed differently between reference frames.

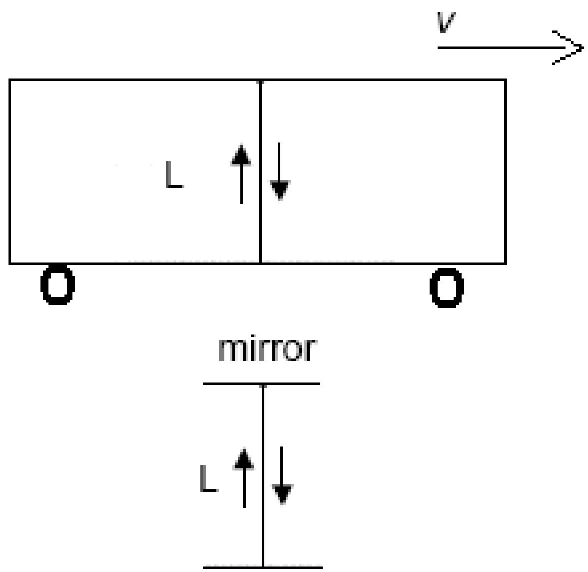
Observed from within Boxcar



Observed from outside Boxcar

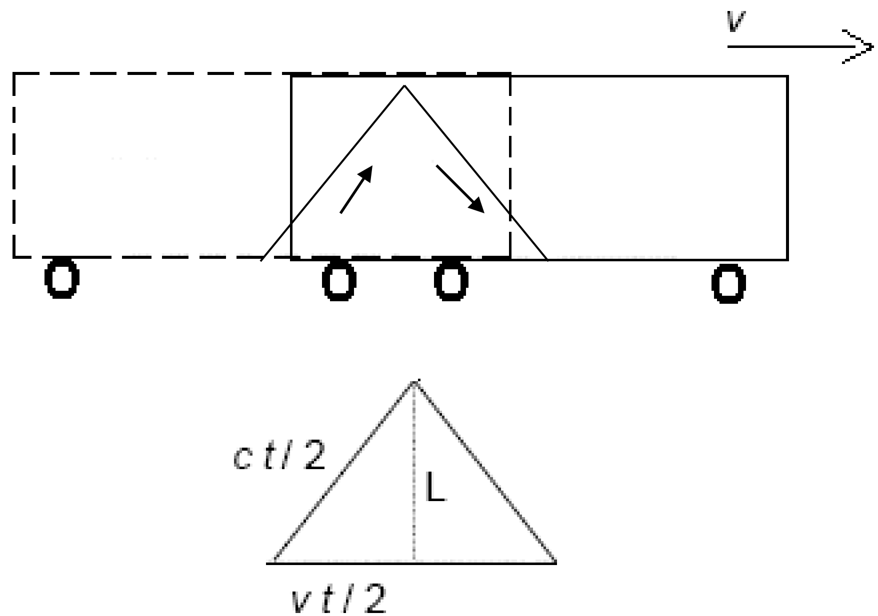


Observed from within Boxcar



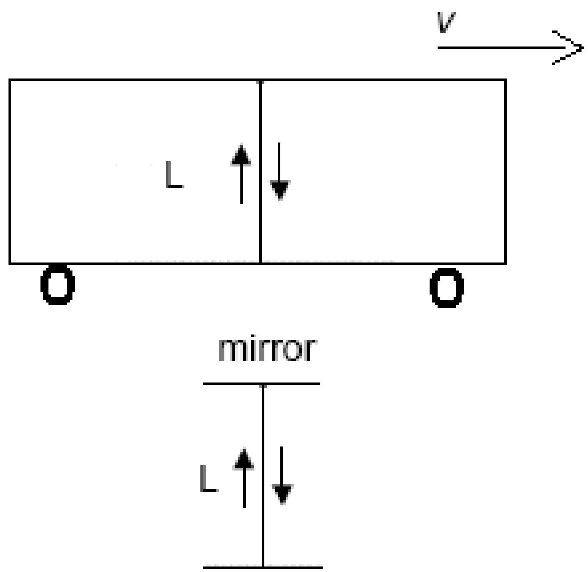
t' = time for light pulse to travel up and return back to boxcar floor, as seen from moving with the box car

Observed from outside Boxcar



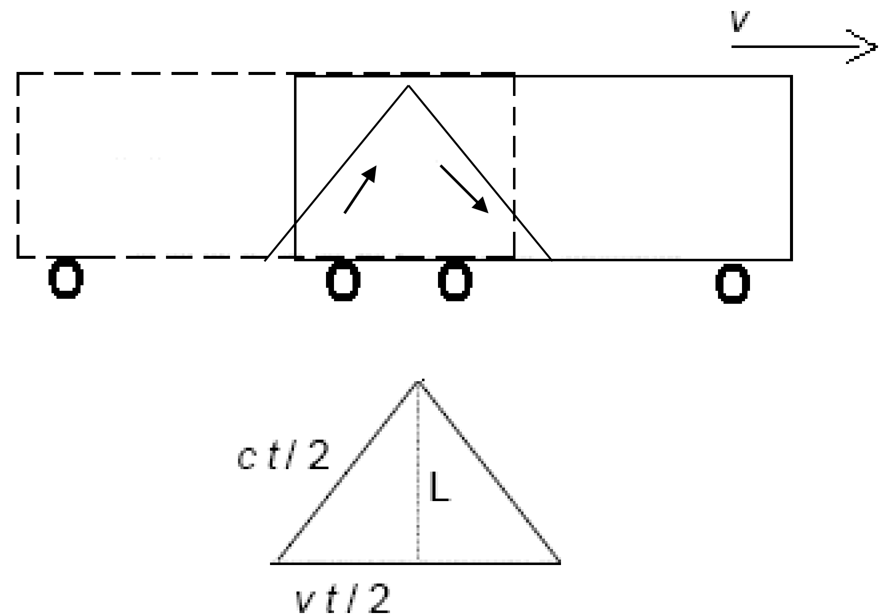
t = time for light pulse to travel up and return back to boxcar floor, as seen from outside boxcar

Observed from within Boxcar



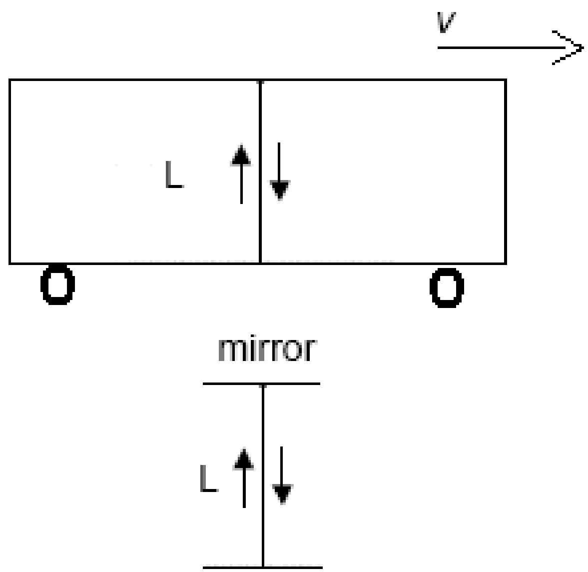
$$2L = ct'$$

Observed from outside Boxcar



$$\left(\frac{vt}{2}\right)^2 + (L)^2 = \left(\frac{ct}{2}\right)^2$$

Observed from within Boxcar



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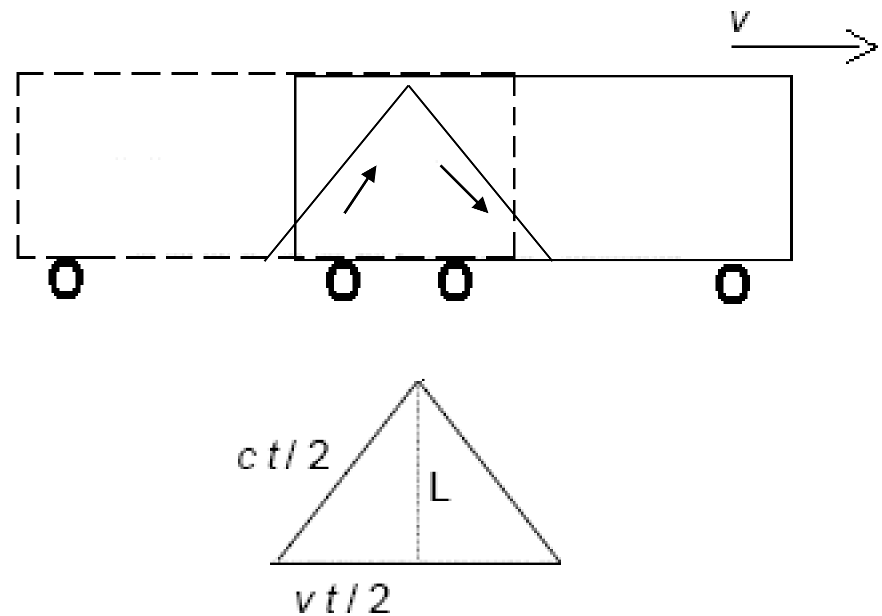
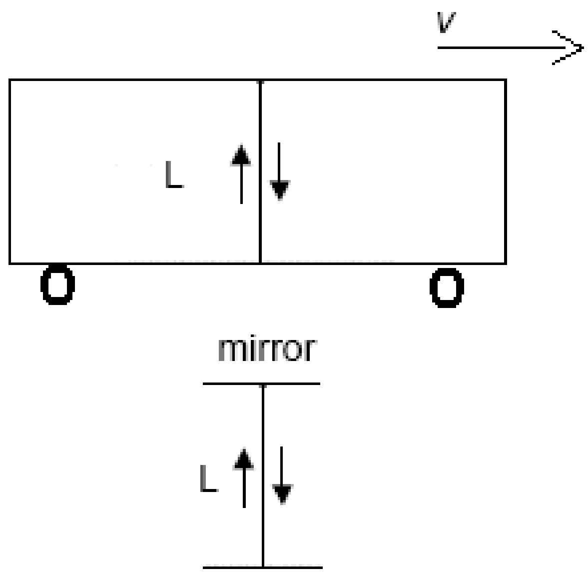


$$\left(\frac{vt}{2}\right)^2 + \left(\frac{ct'}{2}\right)^2 = \left(\frac{ct}{2}\right)^2$$

Observed from within Boxcar



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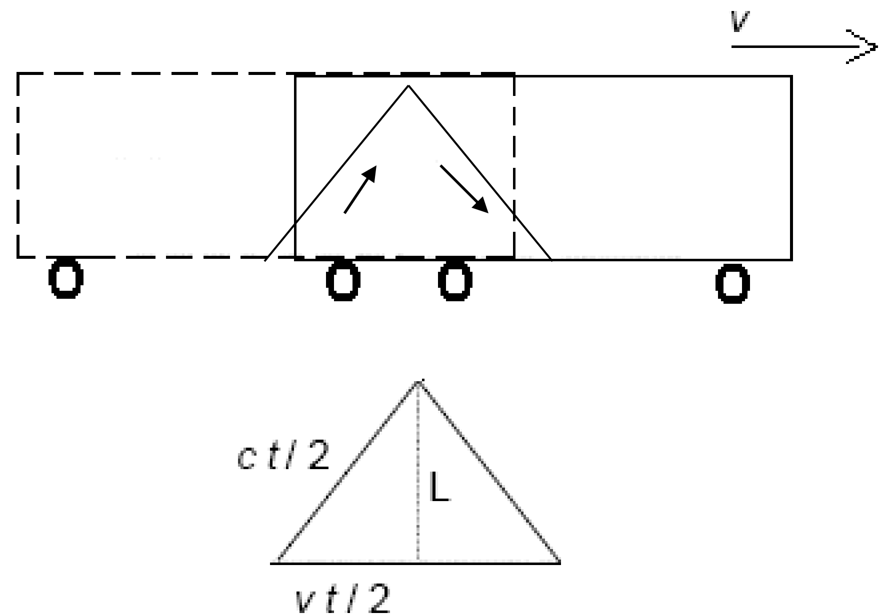


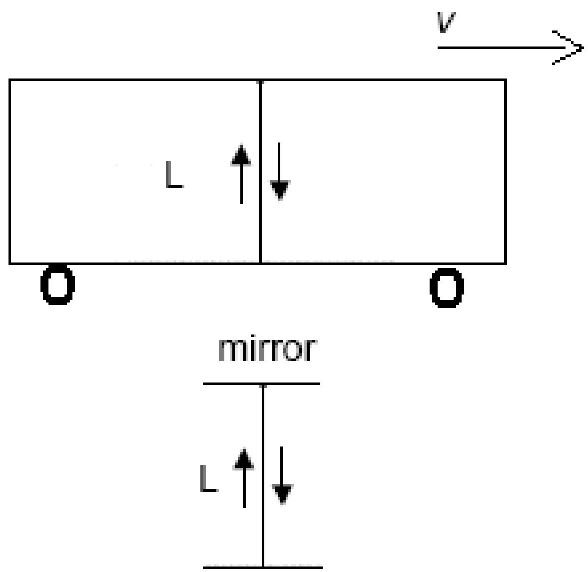
$$2L = ct'$$

$$\left(\frac{vt}{2}\right)^2 + \left(\frac{ct'}{2}\right)^2 = \left(\frac{ct}{2}\right)^2$$

$$\Longrightarrow \quad t' = t\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

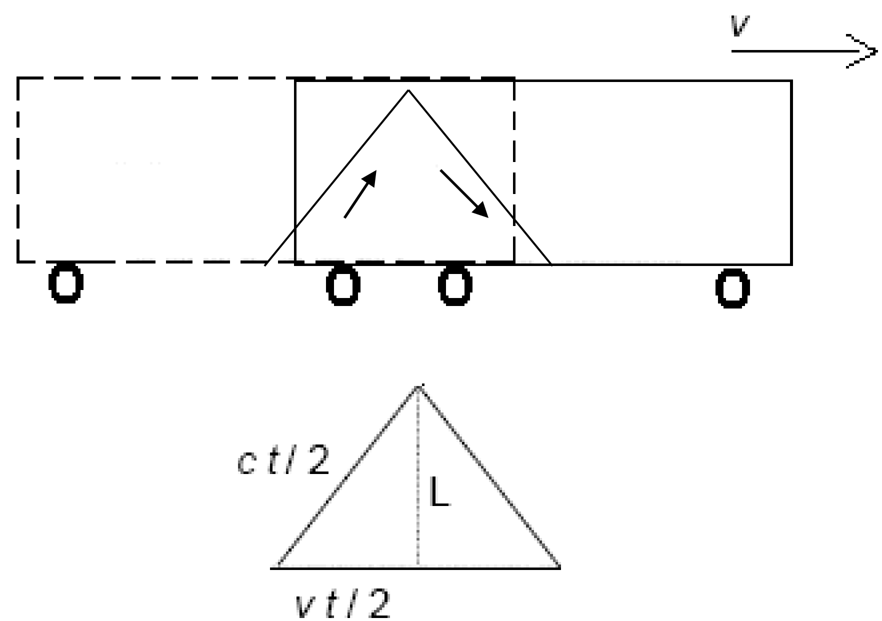
Time dilation

Observed from within Boxcar



t' = time for light pulse to travel up and return back to boxcar floor, as seen from moving <u>with</u> the box car

Observed from outside Boxcar

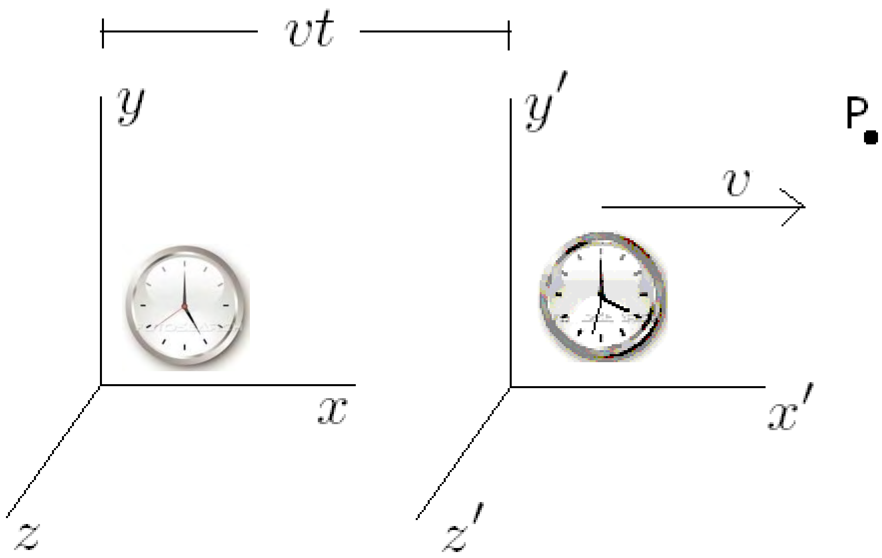


t = time for light pulse to travel up and return back to boxcar floor, as seen from <u>outside</u> boxcar

$$\Longrightarrow \quad t' = t\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

Time dilation

A different space and time transformation

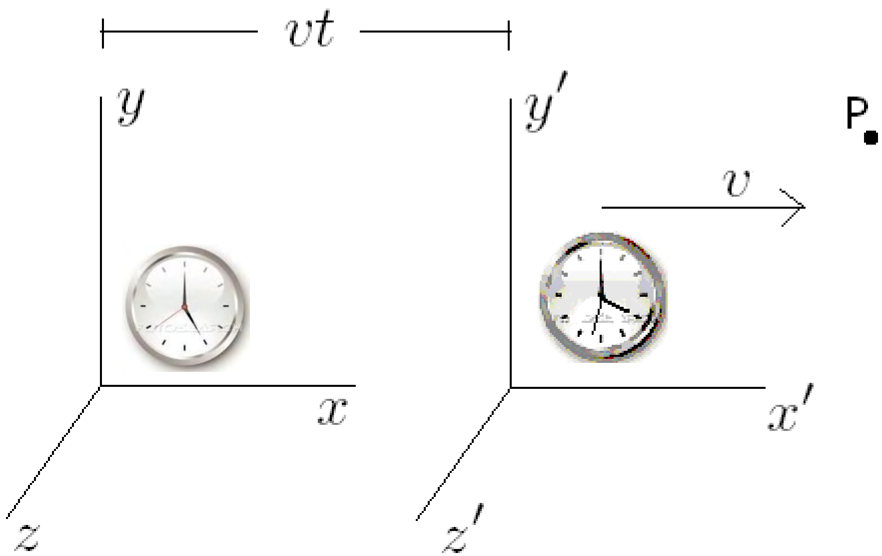


Galilean transformation

$$x' = x - vt$$

$$t' = t$$

A different space and time transformation



$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\gamma > 1$$

Galilean transformation

$$x' = x - vt$$

$$t' = t$$

$$\longrightarrow$$

Einstein's transformation

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

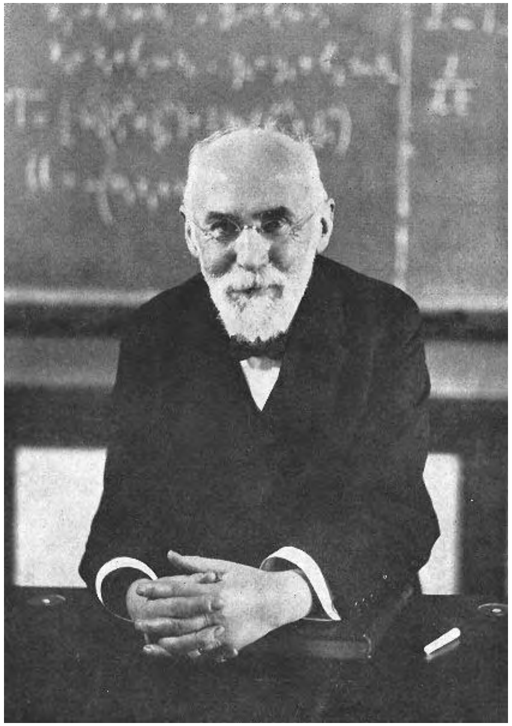
Lorentz Space and Time Transformation

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{v}{c^2}x)$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

H. A. Lorentz



Lorentz sought a transformation in which the equations of electromagnetism would apply to all frames of reference, more as a curious exercise rather than taken as serious reality.

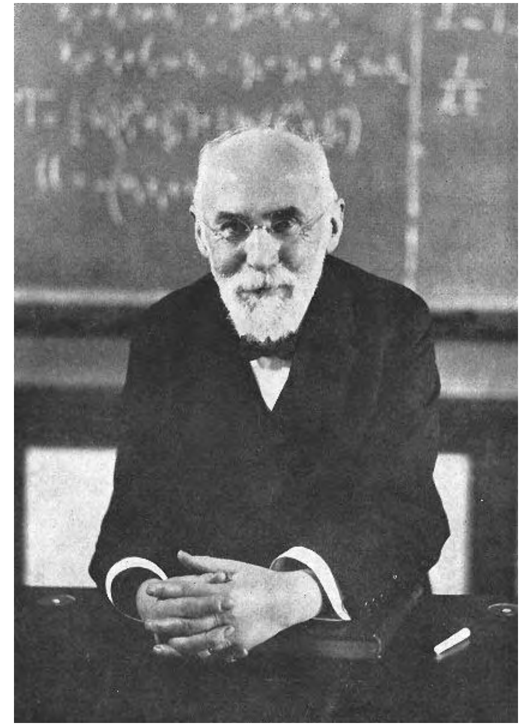
Lorentz Space and Time Transformation

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{v}{c^2}x)$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

H. A. Lorentz



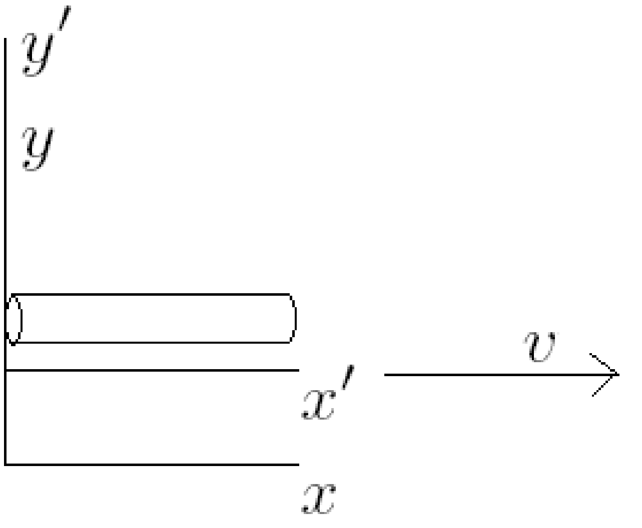
One consequence is the Lorentz contraction, or length contraction:

Lorentz Space and Time Transformation

$$x' = \gamma(x - vt)$$

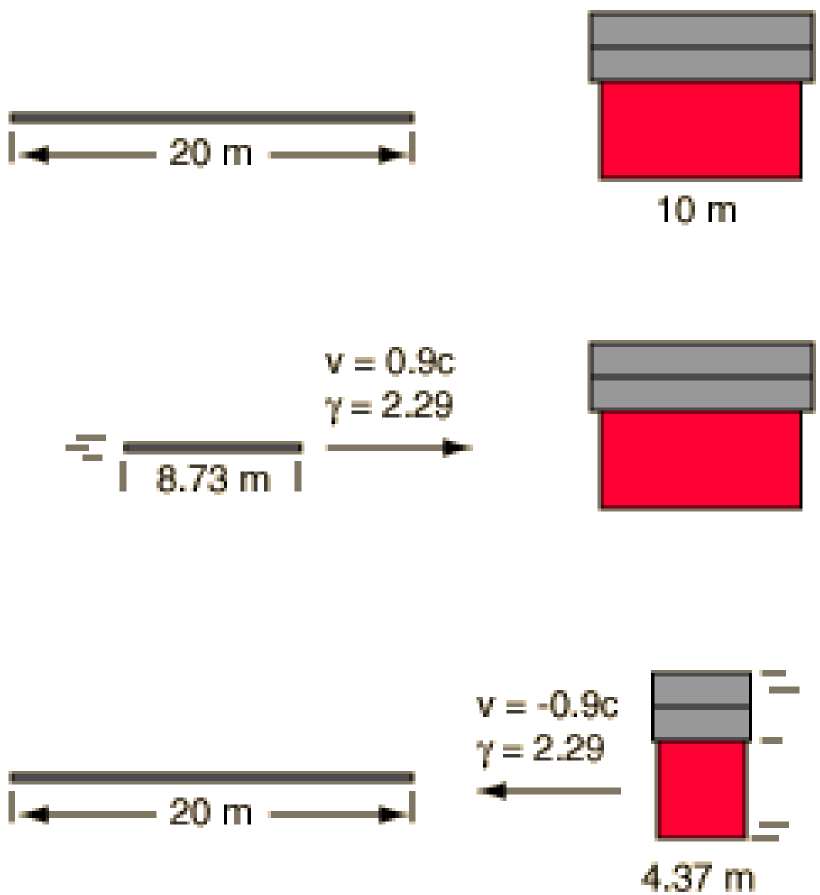
$$t' = \gamma(t - \frac{v}{c^2}x)$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$



One consequence is the Lorentz contraction, or length contraction:

Length contraction and the Barn and pole paradox



A 20 meter pole and a 10 meter barn in their perspective when in the same rest frame.

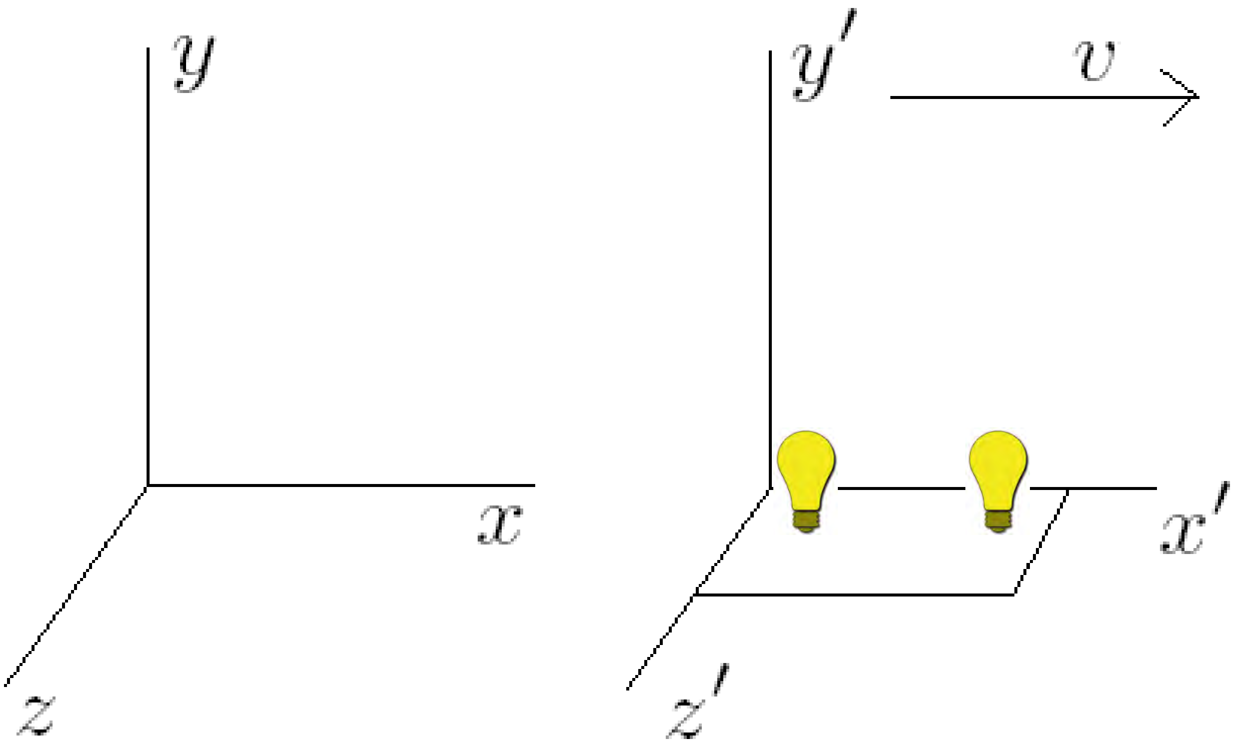
If the pole has speed 0.9c, then it is length-contracted by a factor of 2.29 and short enough to fit momentarily within the barn, at least as seen by an observer in the barn.

But from the point of view of the pole, the barn is contracted and the pole will never fit inside it.

How is it consistent that the moving pole will fit in the momentarily closed barn, as viewed from the pole's reference frame?

The paradox is resolved in view of the relativity of simultaneity

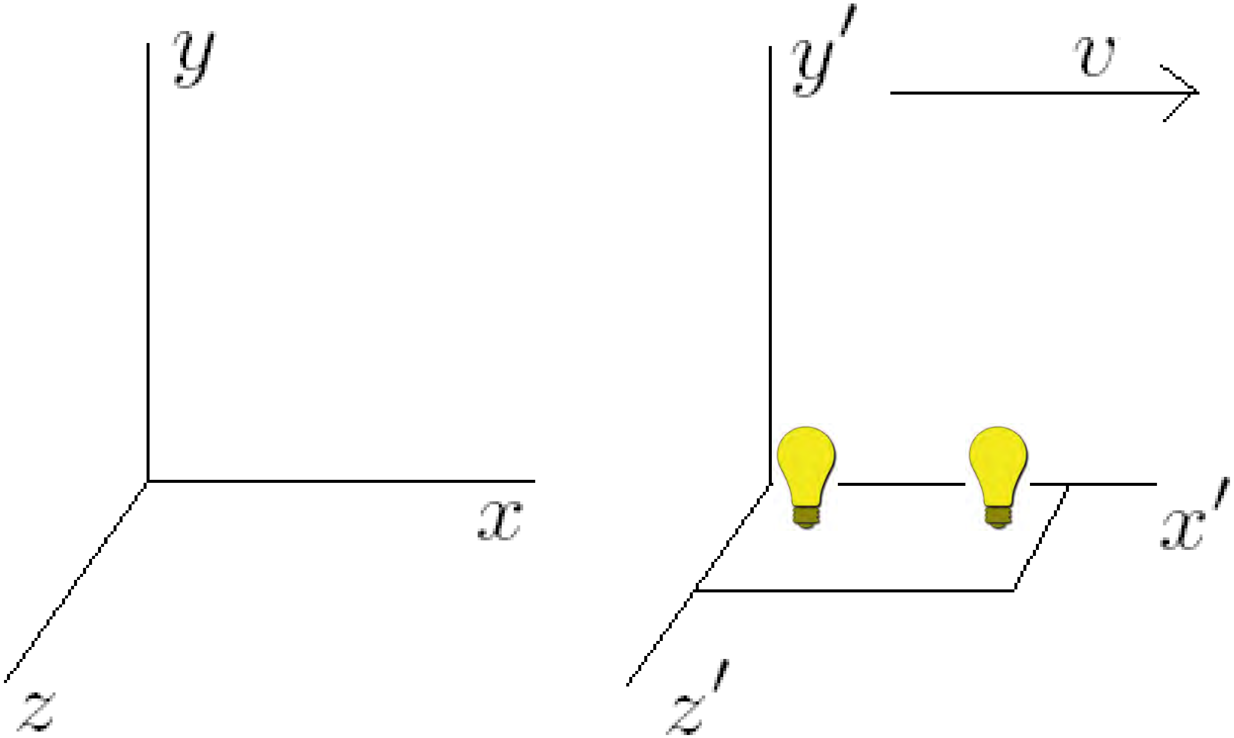
What events are simultaneous is relative!



Relativity predicts that if there is a spatial distance between the moving light bulbs, then if they blink simultaneously as viewed from the primed frame, then they will not blink simultaneously as observed from the unprimed (or any other) inertial frame.

The paradox is resolved in view of the relativity of simultaneity

What are simultaneous events is relative!



$$x' = \gamma(x - vt)$$

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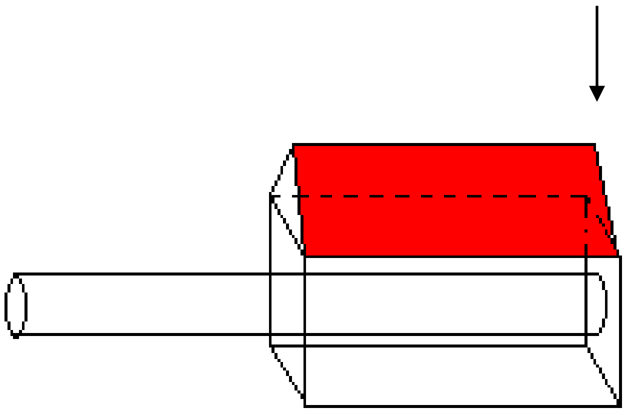
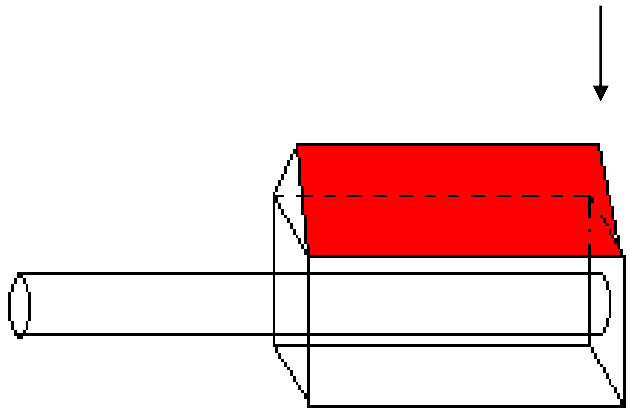
Relativity predicts that if there is a spatial distance between the moving light bulbs, then if they blink simultaneously as viewed from the primed frame, then they will not blink simultaneously as observed from the unprimed frame.

The relativity of simultaneity resolves this paradox!

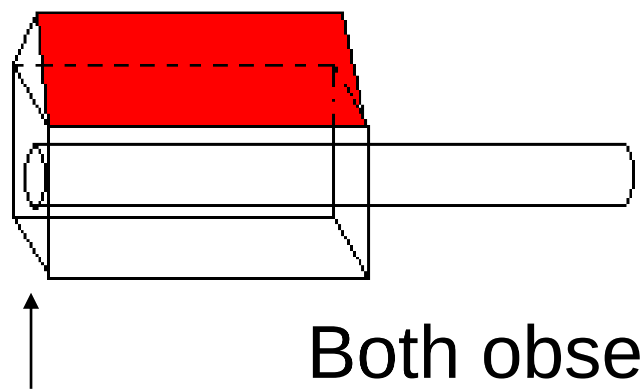


From the pole's frame of reference, <u>first</u> this door momentarily closes, then opens.

The relativity of simultaneity resolves this paradox also!



From the pole’s frame of reference, first this door momentarily closes, then opens.

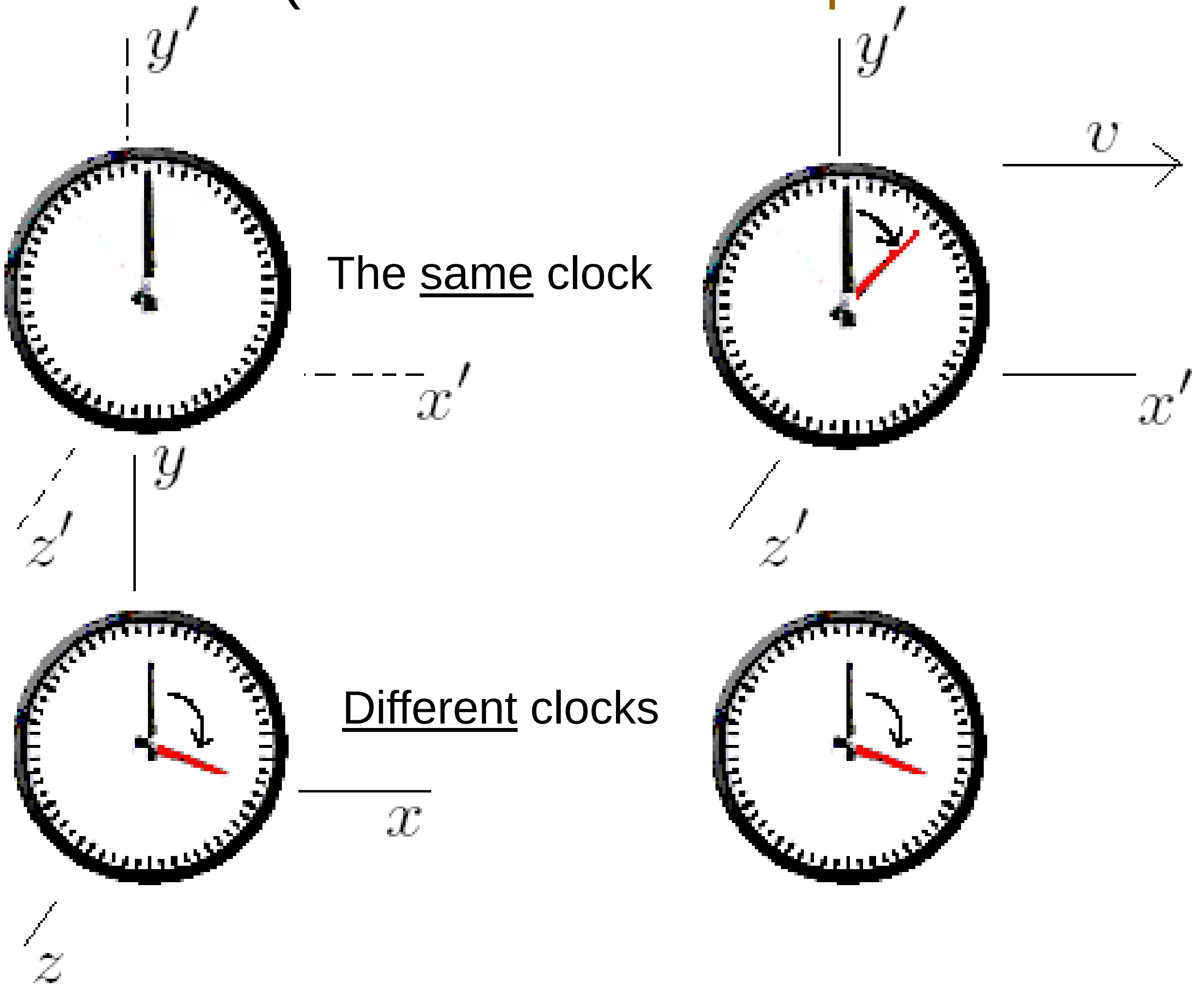


Later, this door closes momentarily, then opens.

Both observers agree the doors closed and opened when an end of the pole was just inside a door.

The Clock Paradox

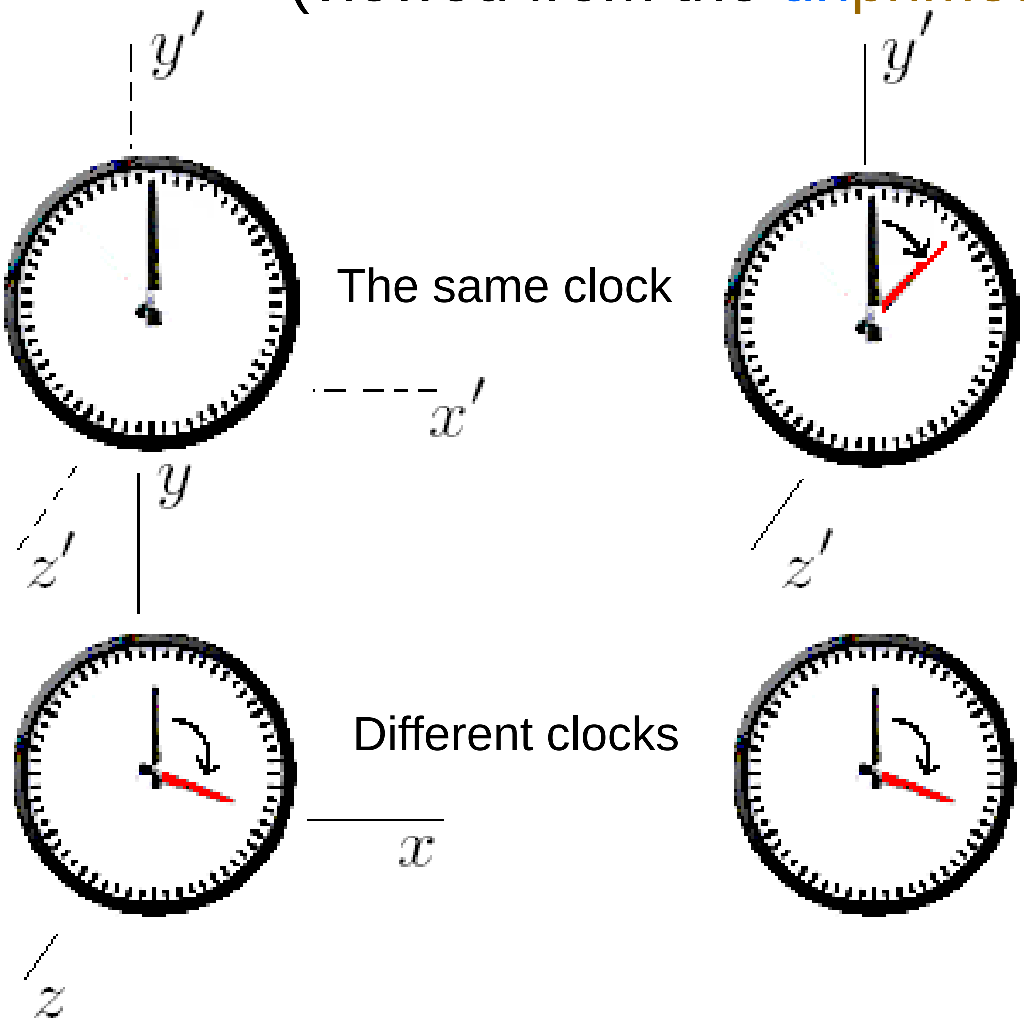
(viewed from the unprimed "stationary" frame)



The red hand is the same hand, just later in time.

The Clock Paradox

(viewed from the unprimed "stationary" frame)

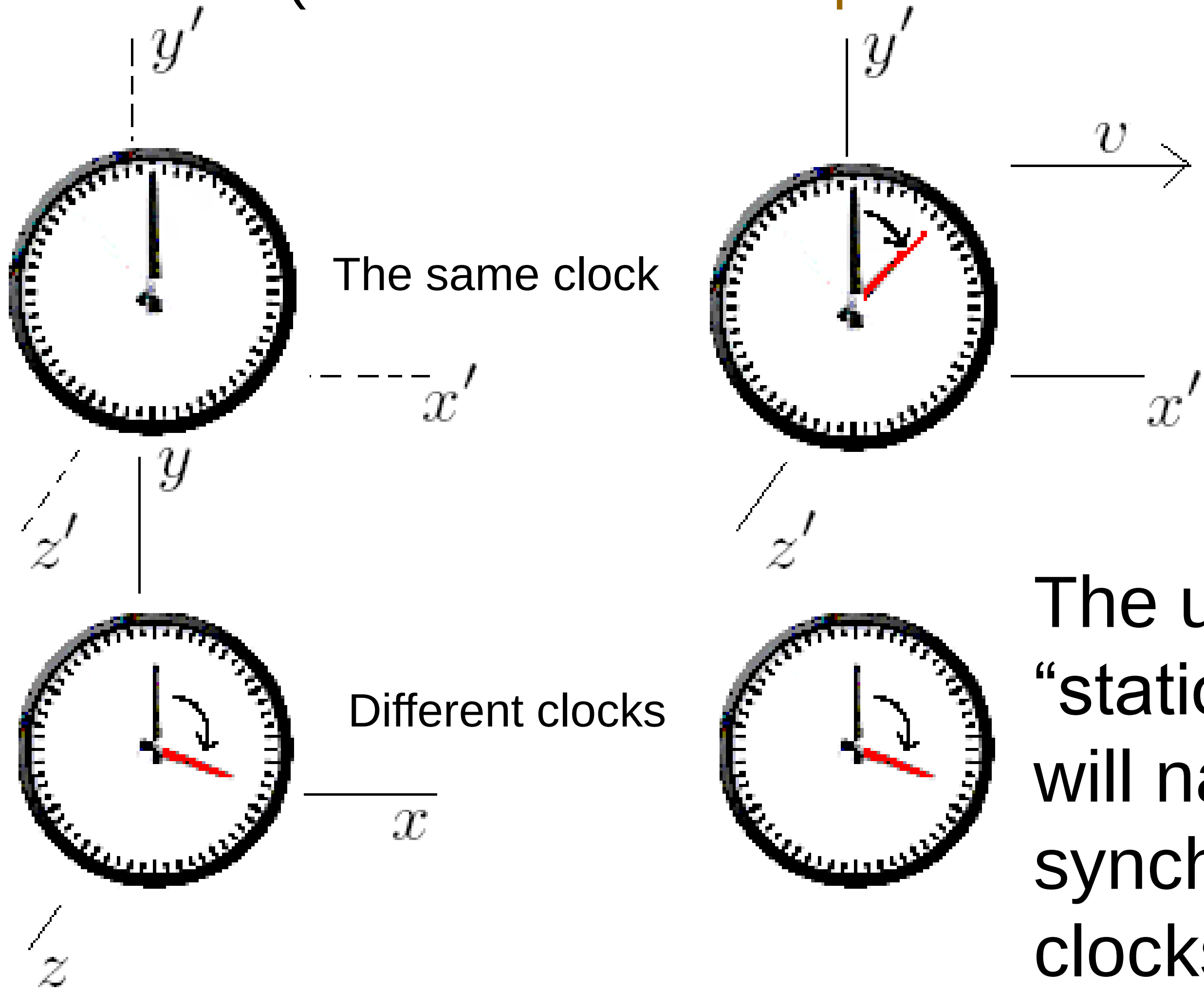


The red hand is the same hand, just later in time.

From the equivalence of inertial reference frames, how can <u>both</u> clocks be running slower compared to the other? Is this not an inconsistency?

The Clock Paradox

(viewed from the unprimed “stationary” frame)

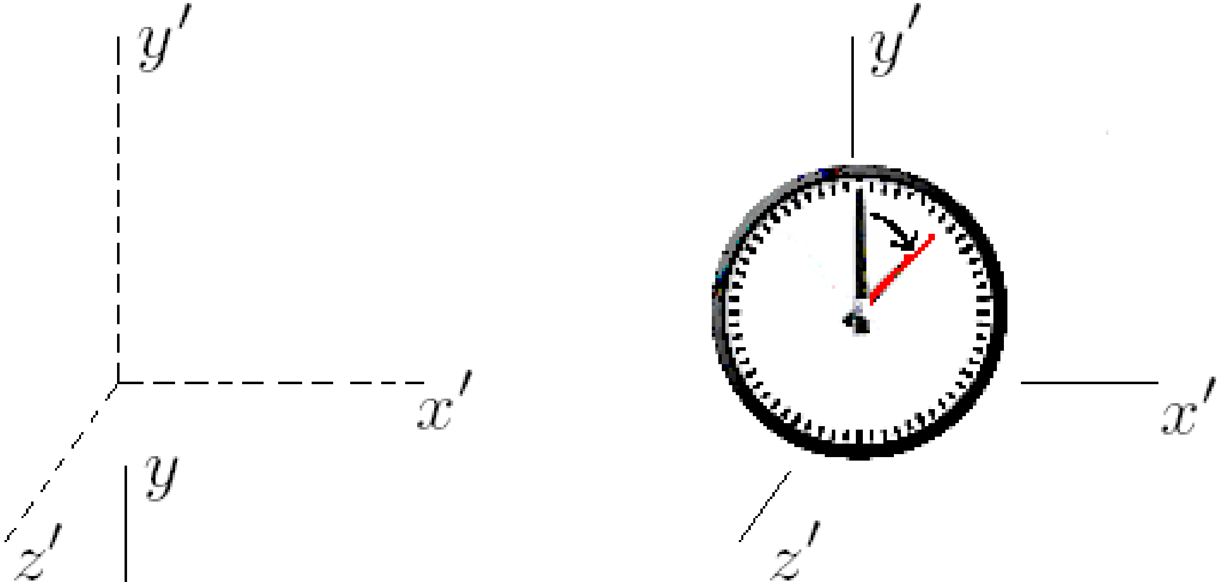


The red hand is the same hand, just later in time.

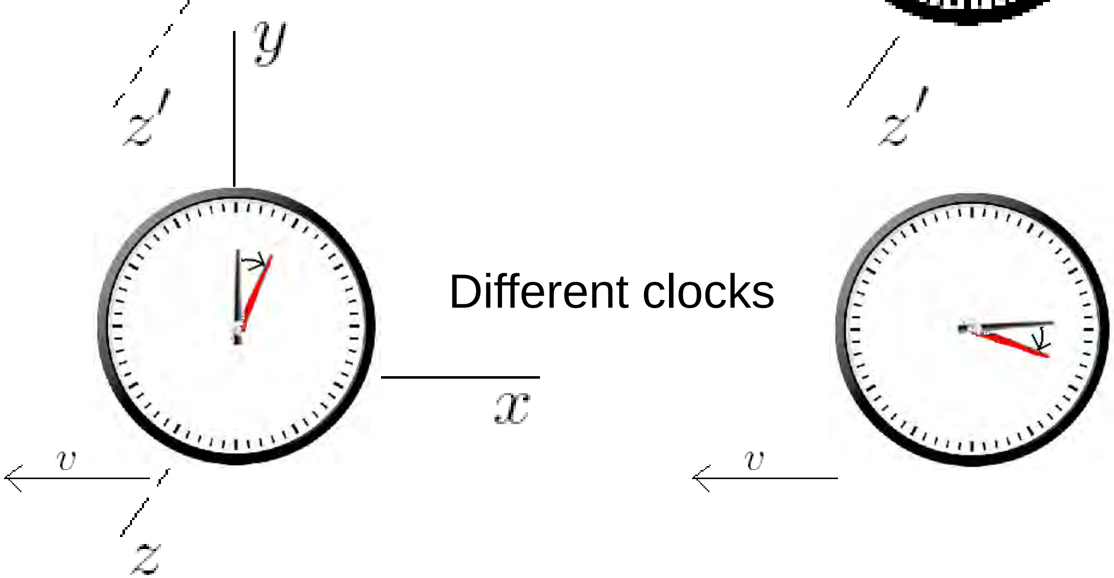
The unprimed “stationary” observer will naturally have synchronized all its clocks.

The Clock Paradox

(viewed from the **primed** “moving” frame)



The red hand is the same hand, just later in time.



However, viewed from the <u>primed</u> “moving” frame, none of the unprimed clocks are synchronized

We are being asked to believe rather startling features of space and time.

Why believe it?

Forget first believing it. First understand the theory, then note experimental evidence.

Experimental Confirmation

- $E= mc^2$ extensively confirmed
  - Nuclear power/bombs
  - Bread and butter of particle accelerator analysis.
- Certain predictions of General Relativity, which is based on Special Relativity, are observed
  - Gravitational lensing, gravitational red shift
  - Aberration in the orbital precession of Mercury
  - Bending of star light
- We see time dilation in particle lifetimes
  - Cosmic ray muons in the upper atmosphere reach the earths' surface, although their rest frame lifetime is too short do so

Decay of Atmospheric Muons